A VERSATILE SPH ALGORITHM FOR MULTIPHASE PIPE FLOW

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I. INTRODUCTION

Multiphase pipe flow is a ubiquitous problem in marine engineering and oil/gas industry. Numerical simulation of multiphase pipe flow faces common challenges such as the large density ratios and complex interfaces. The Smoothed Particle Hydrodynamics (SPH) method has been demonstrated as a promising solution for multiphase flow problems due to its Lagrangian nature. However, its application to multiphase pipe flow may encounter additional issues such as the open boundary condition and numerical instability in turbulent flow state.

The present work aims to establish a versatile algorithm for accurate and stable simulation of multiphase pipe flow. Firstly, the adaptive algorithm is incorporated into the Particle Shifting Technique (PST) to convey more stable recovery of multiphase flows in the framework of weakly compressible SPH (WCSPH). Secondly, the conventional open boundary condition is fine-tuned by novel treatments to guarantee flow continuity in the inflow/outflow regions. Thirdly, the pressure instability and the velocity oscillation emerged in the extreme condition of turbulence is alleviated by the density relaxation algorithm and the modified outflow PST vector.

Validation of the proposed algorithm is carried out through the classic immiscible two-phase co-current flow, presenting appealing agreements with the analytical solutions. Then, the turbulent multiphase pipe flow problems are resolved by the proposed algorithm to showcase its versatility. Robustness of the method is established through turbulent pipe flow cases with high density ratios. These results shed light on the value of the proposed algorithm as an effective solver for complex multiphase flow problems in pipelines.

II. MATERIAL AND METHODS

A. *Weakly compressible SPH model for multiphase flows*

In the multiphase system, the discontinuity of physical properties across the interface may deteriorate numerical stability. Extra diffusions can be introduced as a compensation measure. In this work, we employ the multiphase SPH model proposed in Ref. [1] with the following density correction term to suppress numerical oscillations:

$$D = \eta \nabla^2 \delta\rho \,, \tag{1}$$

where η is the diffusion parameter, and $\delta\rho$ is the density increment calculated from the Equation of State (EoS).

The Particle Shifting Technique [2,3] was initially proposed alleviate unphysical concentration of particles in single-phase simulations. In this work, an adaptive term depending on the specific state of the particle is introduced into the PST algorithm, which allows flexible adjustment of the shifting magnitude and is therefore more justifiable in large density ratio scenario:

$$\delta \boldsymbol{r}_i = -\xi_i \frac{h^2}{2} \Sigma_j [1 + 0.2(\frac{W_{ij}}{W(\delta_0)})^2] \nabla_i W_{ij} V_j \,, \tag{2}$$

where ξ_i is the adaptive parameter depended on the state of the specific particle.

The no-slip boundary condition on pipe walls is realized by the Fixed Ghost Particle technique [4]. It should also be noted that no artificial viscosity is adopted in our simulations.

B. *Inflow/outflow conditions for multiphase pipe flow*

To implement the upstream and downstream flow conditions, we arrange three additional sets of particles (Fig.1): inflow particles, buffer particles and outflow particles.

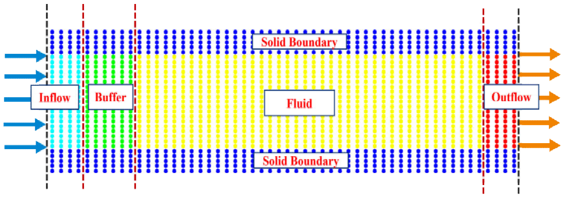


Figure 1. Arrangement of particles in the present SPH simulation of multiphase pipe flows.

The inflow conditions of velocity and pressure are assigned to the inflow particles, and the general periodic initialization condition [5] is imposed to maintain the recycling. For the particles flowing into the buffer region, the velocity relaxation technique [6] is implemented to ensure transition of the candidate particles crossing the buffer threshold into fluid particles. In terms of the outflow region, the fluid particles that cross the outflow threshold will be treated as outflow particles with frozen velocity and density. Once crossing the outlet boundary, the outflow particles will be removed.

In extreme conditions of turbulent pipe flow considered in this work, special treatments should be tailored to ensure numerical stability. First, the pressure fluctuation in the inflow region is difficult to be removed due to the Lagrangian nature of SPH. Such fluctuation may threaten the stability of the inner flow region and even cause massive particle penetrations through solid boundaries. In this work, the following density relaxation algorithm is employed in the buffer region to address this issue:

$$\rho_i = \kappa \rho_i + (1-\kappa)\rho_{ref}\ , \tag{3}$$

where κ is the relaxation factor and ρ_{ref} denotes the reference density of the fluid belonging to a specific phase.

Second, it is found that the velocity instability could be enlarged due to the misuse of the PST algorithm in the outflow region. The truncation of computation domain in the outflow region must be preserved in the PST algorithm. The horizontal component of the PST vector is expected to vanish as the particle is approaching to the outflow boundary. In this work, this expectation is fulfilled by introducing an exponentially diminishing factor to the horizontal component of the PST vector, which gives:

$$\delta \boldsymbol{r}_i^x = \delta \boldsymbol{r}_i^x \cdot \chi \cdot \left(\frac{\left(x_{\max} - r_i^x\right)}{L_{outflow}} \right)^2 , \tag{4}$$

where χ is the ratio of the length of outflow region and the fluid region.

The effectiveness of the above treatments will be detailed in subsection III-B (see Fig. 5 and Fig. 6).

III. RESULTS AND DISCUSSION

A. *Two-phase co-current flow*

To evaluate the accuracy of the proposed algorithm in multiphase problems, an immiscible two-phase co-current flow [7] is simulated (Fig.2). An external force is imposed on either Fluid 1 or Fluid 2 to drive the multiphase system.

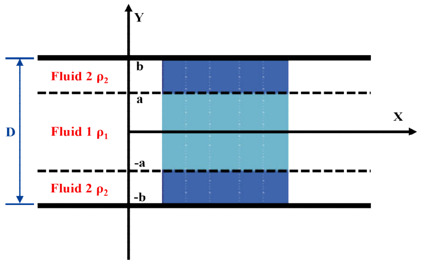
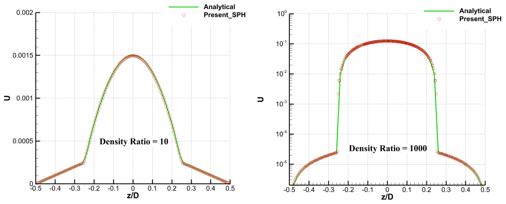
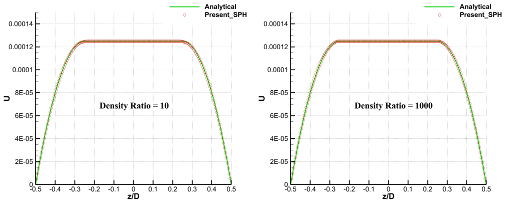


Figure 2. Schematic diagram of immiscible two-phase co-current flow [7].

Fig.3 compares the velocity profiles obtained in our numerical tests and the analytical solutions, in which high density ratio up to 1000 is considered. Good agreements have been achieved in all cases, which demonstrates the accuracy of the proposed algorithm in simulating high-density-ratio multiphase flow in a confined channel.



(a) ρ_2/ρ_1=10 (b) ρ_2/ρ_1=1000



(c) ρ_2/ρ_1=100 (d) ρ_2/ρ_1=1000

Figure 3. Velocity profiles of immiscible two-phase co-current flow ((a) & (b) :external force on fluid 1; (c) & (d) :external force on fluid 2) (the velocity in (b) is shown in the log scale due to the extremely small value)

B. *Turbulent multiphase pipe flow*

For more comprehensive evaluation of the proposed algorithm, turbulent multiphase pipe flows (Fig.4) with different density ratios are simulated in this subsection. In these cases, the Reynolds number is 400,000 with the pipe diameter D=0.2m and the inflow velocity V_{in}=2m/s. The turbulence effect is modeled by the Large Eddy Simulation [8].

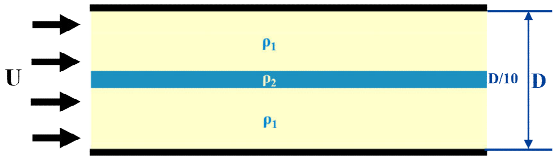
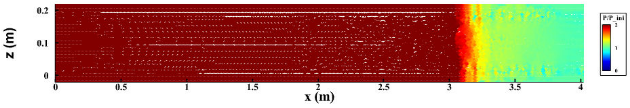


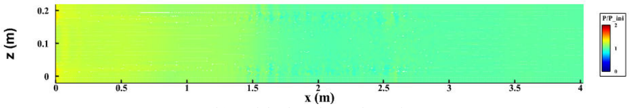
Figure 4. Schematic diagram of turbulent multiphase pipe flow.

Fig.5 presents the effectiveness of the density relaxation algorithm in this extreme flow condition. The comparison of the pressure field proves that the proposed algorithm can significantly suppressed the numerical fluctuation.

Fig.6 (a) and Fig.6 (b) depict a zoom-in view of velocity field of the turbulent multiphase pipe flow, which illustrates that the modification of the PST in the outflow region remedies the anomalous surging of velocity observed in the original simulation.

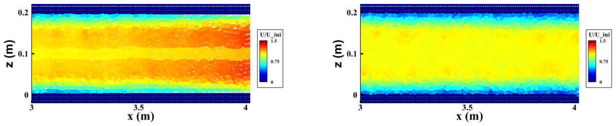


(a) Without density relaxation



(b) With density relaxation

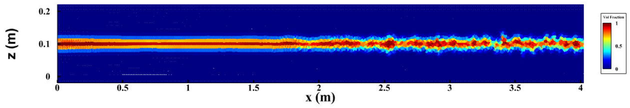
Figure 5. Pressure field of the turbulent multiphase pipe flow obtained by the present algorithm (ρ_2/ρ_1=3).



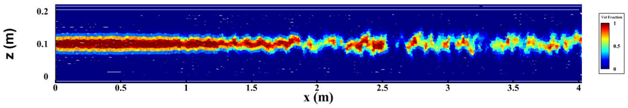
(a) Original PST in the outflow region

(b) Modified PST in the outflow region

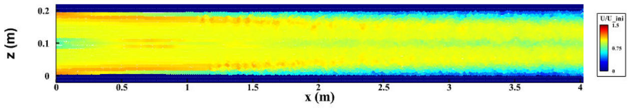
Figure 6. Velocity field of the turbulent multiphase pipe flow obtained by the present algorithm (ρ_2/ρ_1=3).



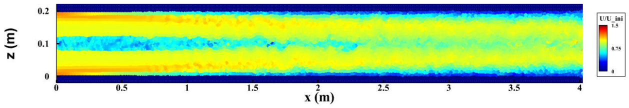
(a) Distribution of volume fraction (ρ_2/ρ_1=1/10)



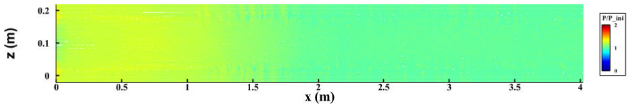
(b) Distribution of volume fraction (ρ_2/ρ_1=1/100)



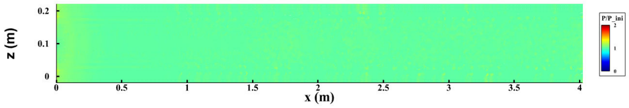
(c) Velocity field (ρ_2/ρ_1=1/10)



(d) Velocity field (ρ_2/ρ_1=1/100)



(e) Pressure field (ρ_2/ρ_1=1/10)



(f) Pressure field (ρ_2/ρ_1=1/100)

Figure 7. Numerical results of turbulent multiphase pipe flow at higher density ratios obtained by the present algorithm.

More numerical results at larger density ratios are given in Fig.7. It is shown in the contours of the volume fraction that the two-phase interaction tends to be more intense as the density ratio increases, which is consistent with physical recognitions. Rational turbulence evolution can be observed along the pipe flow in the velocity field, which demonstrates the capability of the proposed algorithm in recovering turbulent behaviors in multiphase pipe flow. And the contours of the pressure field also show appealing smoothness, which further demonstrates the numerical stability of the proposed algorithm.

IV. CONCLUSION

In this study, a versatile SPH algorithm is developed for effective simulation of complex multiphase pipe flows.

The WCSPH model with modified density diffusion term and magnitude-adaptive PST is employed as the multiphase flow solver. The open boundary condition is fine-tuned by two novel strategies to address numerical instabilities in extreme conditions of turbulent flow: the density relaxation algorithm is proposed to suppress the pressure oscillation in the inflow region, while the PST vector in the outflow region is locally adjusted to stabilize the velocity evolution.

The accuracy of proposed algorithm is validated by the two-phase co-current flow, showing good agreements with the analytical solution. Further tests of turbulent multiphase pipe flow confirm the effectiveness of the novel strategies proposed and the robustness of the algorithm in extreme flow conditions.

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