

δ -SPH stabilization for ISPH scheme

Matthieu DE LEFFE, Corentin HERMANGE, Adrien BOURGOIN, Laurent CHIRON
Siemens Digital Industries Software, Nantes, France
matthieu.de-leffe@siemens.com

I. INTRODUCTION

The SPH method applied to free-surface flows (see [11]) has mainly been developed using the weakly-compressible approach (WCSPH). WCSPH has been successfully used in many applications. However, due to its explicit nature, and the consequent limitation of the time step size, the WCSPH method remains computationally expensive. To overcome this, a semi-implicit incompressible method (ISPH) has been developed (see [4]).

Initial publications with the ISPH scheme encountered the difficulty that the collocated formulation of SPH generates an even/odd decoupling that alters the pressure field (described in [4]). To overcome this, the Laplacian operator in the pressure Poisson equation (PPE) has been replaced by an approximate projection, usually the one proposed by [13]. Later, higher order variants have been proposed (see [8]). Doing so, the principle of virtual work (PVW) is no longer respected within the PPE despite the fact that the importance of this principle in SPH has been highlighted by [2] and [3]. In addition, the intrinsic consideration of the free surface condition which is a strength of the SPH method is no longer valid and must therefore be explicitly imposed. Unfortunately, free-surface detection is a tricky operation in SPH, involving many additional complexities.

Keeping the Laplacian operator that respects the PVW in the PPE and adding stabilization terms is the other possible approach presented in this paper. An additional advantage of this approach is that the intrinsic free surface condition is maintained. Numerous methods exist to stabilize the SPH scheme, a widely used one for WCSPH is δ -SPH ([1][9][10]). An adaptation of this methodology to ISPH is presented in this paper, followed by a validation.

II. SEMI-IMPLICIT ISPH

The basic of ISPH scheme is briefly presented to introduce the notations. The reader can refer to these authors for further details on the origin of the ISPH scheme: [4], [5], [7], [8] and [14].

First, a position prediction of particles is performed according to the velocity at the previous iteration:

$$\mathbf{r}_i^* = \mathbf{r}_i^n + \Delta t \mathbf{u}_i^n \quad (1)$$

with \mathbf{r} the particle position, Δt the time step and \mathbf{u} the particle velocity. Secondly, a velocity prediction step is performed, based on the new position field. The momentum equation reads:

$$\mathbf{u}_i^* = \mathbf{u}_i^n + \Delta t \nu \mathbf{L}\{\mathbf{u}^n\}_i + \Delta t \mathbf{g} \quad (2)$$

with \mathbf{g} is the gravity acceleration and ν the fluid kinematic viscosity. The discrete Laplacian operator \mathbf{L} used for the viscous force is the one proposed by [12]. The pressure is then computed from the PPE:

$$D^-\{\mathbf{G}^+\{p^{n+1}\}\}_i = \frac{\rho}{\Delta t} D^-\{\mathbf{u}^*\}_i \quad (3)$$

with ρ the fluid density and p the particle pressure. Note that the $D^-\{\mathbf{G}^+\}$ operator is preserved in (3) and will be discussed in the following chapter. Indeed, to prevent the even/odd decoupling of the pressure field, [4], [5], [7], [8] or [14] replace this $D^-\{\mathbf{G}^+\}$ operator by the Laplacian proposed by [13] or more accurate variants. The gradient and divergence operators are defined by the adjoint operators:

$$\mathbf{G}^+\{p\}_i = \sum_j \omega_j (p_j + p_i) \nabla W_{ij} \quad (4)$$

$$D^-\{\mathbf{u}\}_i = \sum_j \omega_j (\mathbf{u}_j - \mathbf{u}_i) \cdot \nabla W_{ij} \quad (5)$$

with ω the particle volume. These adjoint operators lead to energy-conserving properties and does not require the introduction of a free surface condition, cf. [3]. The velocity is thereafter corrected from the pressure gradient:

$$\mathbf{u}_i^{n+1} = \mathbf{u}_i^* - \frac{1}{\rho} \mathbf{G}^+\{p^{n+1}\}_i \quad (6)$$

And finally, the position is updated from the new velocity:

$$\mathbf{r}_i^{n+1} = \mathbf{r}_i^* + \Delta t (\mathbf{u}_i^{n+1} - \mathbf{u}_i^n) \quad (7)$$

A shifting step can then be carried out, but this will not be studied here, as δ -ISPH does not necessarily need it. More accurate time schemes can also be used, but for convenience they will not be discussed here.

III. δ -ISPH SCHEME

The δ -SPH scheme is introduced in the semi-implicit ISPH scheme as follows. First a diffusive term is introduced in the momentum equation, so the velocity prediction (2) is modified:

$$\mathbf{u}_i^* = \mathbf{u}_i^n + \Delta t \nu \mathbf{L}\{\mathbf{u}^n\}_i + \Delta t \mathbf{g} + \alpha \sigma \sum_j \omega_j \tilde{u}_{ij} \pi_{ij} \nabla W_{ij} \quad (8)$$

with α a coefficient controlling the intensity of the artificial viscosity, σ the kernel standard deviation defined in [16], and

\tilde{u}_{ij} a reference velocity scale. In WCSPH, the speed of sound is used as the reference velocity. In ISPH, based on [16] by studying the stability of the scheme, an appropriate value is:

$$\tilde{u}_{ij} = \sqrt{\frac{|p_i| + |p_j|}{2\rho}} \quad (9)$$

The absolute value avoids a non-definition of the reference velocity specifically at the free surface, but other choices may be relevant. Finally, π_{ij} is identical to [9].

$$\pi_{ij} = \frac{(\mathbf{u}_j^n - \mathbf{u}_i^n) \cdot \mathbf{r}_{ji}^*}{\|\mathbf{r}_{ij}^*\|^2} \quad (10)$$

An additional artificial diffusive term is also used into the continuity equation, which counteracts the checkerboard effect on pressure. The PPE (3) is modified as following:

$$D^-\{\mathbf{G}^+\{p^{n+1}\}\}_i = \frac{\rho}{\Delta t} D^-\{\mathbf{u}^*\}_i - \delta \sum_j \omega_j \psi_{ij} \cdot \nabla W_{ij} \quad (11)$$

with δ a coefficient controlling the intensity of the diffusion, and where

$$\psi_{ij} = 2(p_j^{n+1} - p_i^{n+1}) \frac{\mathbf{r}_{ji}}{|\mathbf{r}_{ij}|^2} - [\mathbf{G}^-\{p^{n+1}\}_i + \mathbf{G}^-\{p^{n+1}\}_j] \quad (12)$$

Note that in [9] ψ_{ij} is computed from the density. Since in [9] the equation of state used is linear, it is possible to replace density by pressure and find the relation presented here. Note also that the second term uses a non-renormalized gradient of pressure. According to our tests, renormalizing brings no noticeable improvement.

These modifications have no impact on the free surface condition. The δ -ISPH model presented here therefore requires no additional condition at the free surface as for WCSPH.

IV. RESULTS

The SPHERIC benchmark case n°2 is studied to analyse the behaviour of the δ -ISPH model. This case is a dam-break flow with 3D effects where accurate experimental pressure measurements are available, cf [6]. This problem involves a rectangular step placed in a tank, in front of an initial fluid domain.

The Figure 1 and Figure 2 show the pressure field in a vertical section in the middle of the flow for a coarse discretization ($H/\Delta x=32$, H the initial water height, Δx initial particle spacing). The chessboard effect is clearly visible in the lower part of the flow in Figure 1. The Figure 2 using the δ -ISPH model illustrates the contribution of the model by providing a continuous pressure field.

In the experiment, eight pressure probes were placed along the middle section of the step. In Figure 3 the pressure signals on P₃ probe predicted by the δ -ISPH model are displayed compared to ISPH without stabilization, and experiment, for a

middle discretization ($H/\Delta x=64$). The result obtained with δ -ISPH is much less noisy in time, a corollary of the less spatially noisy pressure field, and closer to the experiment. However, the value predicted by δ -ISPH is higher from 1s to 2s. A convergence study with a finer discretization, $H/\Delta x=128$, did not improve this result. A time lag is also visible on the wave return at around 5s. This difference remains to be studied in further detail.

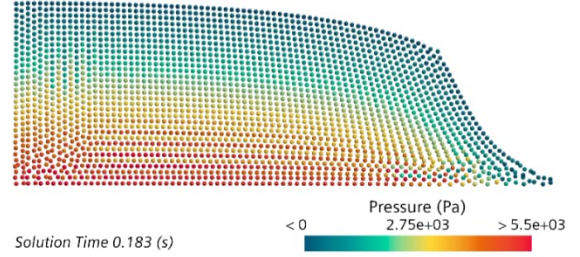


Figure 1: Pressure field in the middle vertical section without stabilization

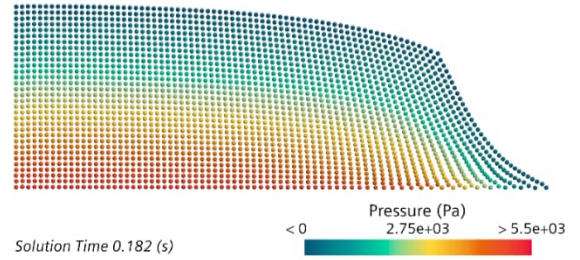


Figure 2: Pressure field in the middle vertical section with stabilization

In the experiment, water heights have also been measured. In Figure 4 the water height signals on H₂ probe predicted by the δ -ISPH model are displayed compared to ISPH without stabilization, and experiment. For water height, there is a very good agreement between δ -ISPH, ISPH and experiment. The time lag is still visible on the wave return at around 5s. This good agreement between δ -ISPH and ISPH is probably since δ -ISPH has no effect on overall flow behaviour, only on the quality of the pressure field.

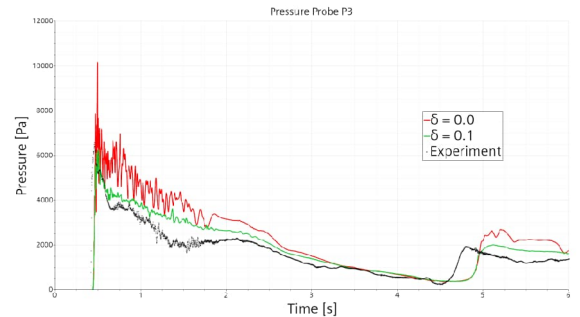


Figure 3: Comparison of the pressure signals predicted at probe P₃ for the scheme with and without stabilization, with the one recorded experimentally

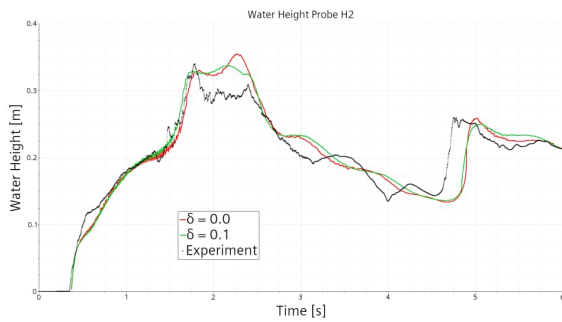


Figure 4: Comparison of the water height predicted at probe H_2 for the scheme with and without stabilization, with the one recorded experimentally

The Figure 5 shows the velocity field at 0.9s, i.e. shortly after impact on the step, for a fine discretization ($H/\Delta x=128$). The free surface is remarkably well maintained. This is achieved thanks to the free surface treatment, similar to that used in WCSPH. In contrast, by imposing zero pressure on the free surface in the usual ISPH methods, the cohesion of the free surface is degraded and requires additional treatment.

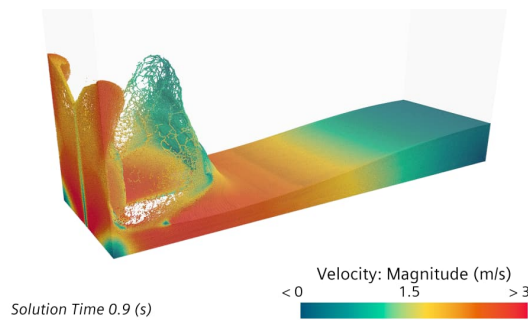


Figure 5: Velocity field at $t=0.9s$ for the fine discretization

From a computation time point of view, the $D^{-}\{\mathbf{G}^+\}$ operator implies a double loop and therefore a larger neighbourhood than the approximate operator [13]. However, with the δ -ISPH model the linear system of the PPE is faster to converge using an iterative method, since for the same target error, the $D^{-}\{\mathbf{G}^+\}$ operator requires on average 30% fewer iterations on this dam break case. This seems counterintuitive as the matrix is less diagonal dominant. There is probably a link with the use of adjoint operators, but this remains to be clarified. In addition, δ -ISPH eliminates the extra computation to search for free surfaces.

Finally, δ -ISPH does not suffer from strong volume conservation issues, probably thanks to the absence of shifting method, which is mandatory when using the operator [13].

V. CONCLUSIONS

An δ -ISPH model based on δ -SPH stabilization has been presented. This model offers significant advantages compare to usual ISPH model [4]:

- Use of adjoint operators and therefore the free surface condition intrinsic to the SPH method is preserved.

- The scheme works without shifting, which can be added optionally.
 - δ -ISPH is much more robust especially in free-surface areas.
- The comparison in terms of computation time with usual ISPH model remains to be clarified and will be the subject of future work.

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