SPH modelling of water flow inside a porous medium using a Riemann based formulation

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I. Introduction

Simulating granular flows has been a major concern for the past decades, with various environmental applications such as avalanches, landslides and scour near fluvial, coastal and off-shore constructions. These phenomena involve a porous matrix undergoing large deformations and characterized by a complex behaviour. Thanks to its ability to handle large deformations of the medium, SPH appears as a particularly advantageous method to deal with these problems. Various SPH schemes already exist to model the granular medium behaviour, such as the ones developed by Feng *et al.* [1], Bui and Nguyen [2] (elastoplastic approach) or Ghaïtanellis [3] (elasto-viscoplastic approach). However, to the authors' knowledge, none of these papers show in their results what the water flow looks like inside the porous structure while it deforms. Consequently, the present work aims at presenting a model to simulate the deformation of a granular medium as well as its infiltration by water. The porous structure is unsaturated and is considered as a continuous medium. Infiltrated water flows according to Darcy's or Darcy-Forchheimer's laws. Two different sets of particles are used for this purpose, one for the water phase and an other one for the porous structure, as done by Shimizu *et al* [4]. Riemann-based schemes have never been used yet for such purpose, though it proved to give accurate results for both fluid [6] and solid mechanics [7]. As a consequence, a new approach is proposed here by applying Riemann-based formulations on both fluid and solid media. For the moment, the proposed model is not yet finished developing and only simulations with undeformable porous matrix are available for the writing of this abstract. Results with moving, deformable porous matrix will be presented during the conference.

II. Governing equations

As mentionned earlier, the porous matrix is considered undeformable for the moment. Therefore, the following set of equations constituting our model only describes the motion of the fluid infiltrating the structure:

$$\begin{cases} \dfrac{D\boldsymbol{x}}{Dt} &= \boldsymbol{U} & (1) \\ \dfrac{D\rho}{Dt} &= -\rho\, div\, \boldsymbol{U} - \dfrac{\rho}{\varphi}\boldsymbol{U}\cdot\boldsymbol{grad}\,\varphi & (2) \\ \dfrac{D\boldsymbol{U}}{Dt} &= -\dfrac{1}{\rho}\boldsymbol{grad}\,p & \\ & \quad + \dfrac{1}{\rho\varphi}\boldsymbol{div}(\varphi\mu\,[\mathbb{G}rad\,\boldsymbol{U} + \mathbb{G}rad\,\boldsymbol{U}^T]) & \\ & \quad + \boldsymbol{g} + \dfrac{1}{\rho\varphi}(\boldsymbol{R}_D + \boldsymbol{R}_F) & (3) \\ p &= c_0^2(\rho - \rho_0) & (4) \end{cases}$$

where :

- $\boldsymbol{x}$, $\boldsymbol{U}$, ρ and p are the fluid position, velocity, density and pressure fields, respectively.
- φ is the fluid volume fraction. It represents the proportion of water present in a representative elementary volume of the domain.
- μ is the dynamic viscosity of water and $\boldsymbol{g}$ is the gravity.
- $\boldsymbol{R}_D$ and $\boldsymbol{R}_F$ refer to Darcy's and Forchheimer's resistance terms, respectively. They govern the water flow inside the porous domain, and are expressed as follows:

$$\boldsymbol{R}_D = -\frac{\mu}{K_p}\varphi^2\boldsymbol{U} \quad , \quad \boldsymbol{R}_F = -\frac{F_{ch}\rho}{\sqrt{K_p}}\,\varphi^3\,||\boldsymbol{U}||\,\boldsymbol{U} \qquad (5)$$

with K_p being the matrix hydraulic permeability and F_{ch} the Forchheimer's coefficient. Their expressions depends on D_c, the average diameter of the grains, here taken equal to $0.0159\,m$, and β, a numerical parameter set to 150:

$$K_p = \frac{\varphi^3 D_c^2}{\beta(1-\varphi)^2} \quad , \quad F_{ch} = \frac{1.75}{\sqrt{\beta\varphi^3}} \qquad (6)$$

- c_0 and ρ_0 are the numerical speed of sound and the reference density of the fluid, respectively. A weakly compressible approach is chosen in this work, resulting in slight variations of the fluid density around ρ_0 provided that the value of c_0 is chosen large enough with respect to the fluid velocity.

III. Numerical model

The domain is discretized using two separate sets of particles: one for the fluid, and another one for the structure. Equations (1)-(4) are discretized onto the fluid particles set, using a Riemann solver based on the work of Vila [9] and Parshikov and Medin [8]. For a fluid particle i, the model reads:

$$\begin{cases} \dfrac{D\boldsymbol{x}_i}{Dt} &= \boldsymbol{U}_i + \boldsymbol{\delta U}_i & (7) \\ \dfrac{D\rho_i}{Dt} &= -\rho_i\left(\langle div\,\boldsymbol{U}\rangle_i^{BIM} + \Theta_i^{div\,\boldsymbol{U}}\right) & \\ & \quad - \dfrac{\rho_i}{\varphi_i}\boldsymbol{U}_i \cdot \langle \boldsymbol{grad}\,\varphi\rangle_i^{BIM} & (8) \\ \dfrac{D\boldsymbol{U}_i}{Dt} &= -\dfrac{1}{\rho_i}\left(\langle \boldsymbol{grad}\,p\rangle_i^{BIM} + \Theta_i^{\boldsymbol{grad}\,p}\right) & \\ & \quad + \dfrac{1}{\rho_i\varphi_i}\langle \boldsymbol{lap}\,\boldsymbol{U}\rangle_i^{BIM} & \\ & \quad + \boldsymbol{g} + \dfrac{1}{\rho_i\varphi_i}(\boldsymbol{R}_{D_i} + \boldsymbol{R}_{F_i}) & (9) \\ p_i &= c_0^2(\rho_i - \rho_0) & (10) \end{cases}$$

where $\langle\bullet\rangle$ refer to SPH operators in Boundary Integral Method (BIM) formalism as detailed in Chiron *et al.* [10]. Note that structure particles are excluded from the computation of these operators. φ_i is the particle volume fraction. The evolution of this quantity is not govern by any differential equation. Its calculation lies on a local saturation condition and is achieved using an SPH interpolation as the one done by Shimizu *et al.* [4]:

$$\varphi_i = 1 - \frac{1}{\gamma_i}\sum_{j\in\Omega_i^s}\varphi_j W_{ij} V_j \qquad (11)$$

where γ_i is a wall renormalization factor [12], Ω_i^s is the kernel support of the particle i considering only its solid neighbouring particles, and W denotes the SPH kernel.

A particle shifting technique is used through the term $\boldsymbol{\delta U}$ as in Michel *et al.* [5]. Finally, $\Theta_i^{div\,\boldsymbol{U}}$ and $\Theta_i^{\boldsymbol{grad}\,p}$ are diffusion terms expressed as:

$$\Theta_i^{div\,\boldsymbol{U}} = \sum_{j\in\Omega_i^f}(2\,U_{ij}^{*N}\boldsymbol{n}_{ij} - \boldsymbol{U}_i - \boldsymbol{U}_j)\cdot\boldsymbol{\nabla}W_{ij}V_j \qquad (12)$$

$$\Theta_i^{\boldsymbol{grad}\,p} = \sum_{j\in\Omega_i^f}(2\,p_{ij}^* - p_i - p_j)\boldsymbol{\nabla}W_{ij}V_j \qquad (13)$$

where U_{ij}^{*N} and p_{ij}^* are the solutions of the Riemann problem at the virtual interface between i and j, and $\boldsymbol{n}_{ij}$ is the unit normal at this interface. To increase the precision of the scheme, MUSCL reconstructions are applied.

IV. Model validation

The performance of the presented model is now tested against two 2D benchmarks with undeformable porous structure.

First, our model is validated on the 2D water dambreak with a porous obstacle benchmark. The initial setup consists of a tank containing a porous matrix located at its center, with a porosity $p_w = 0.49$. On the left of the matrix, a water column is retained by a gate. Finally, a low water bed is present in the rest of the tank. Fig 1 shows the pressure field obtained at two different time instants, as well as a comparison between our present free-surface solution and the one from Shimizu *et al.* [4] numerical model (ISPH formalism) and Liu *et al.* [11] experiment. As it can be seen, the free-surface obtained with our model is in good agreement with results found in the literature, and the pressure field obtained is quite smooth.

The second benchmark chosen to validate the proposed model is the 2D U-tube with porous obstacle. Here, a square structure with a porosity $p_w = 0.4$ is located at the center of a U-shaped tube with an initial water level difference between its left and right vertical sections, causing the water to seep through the porous matrix. For this benchmark, the resistance force is written quite differently. The first term corresponding to Darcy's law, $\boldsymbol{R}_D$, is no longer written as in (5) but:

$$\boldsymbol{R}_D = -\frac{1-\varphi}{1-p_w}\frac{\rho g}{K_h}\varphi^2\boldsymbol{U} \qquad (14)$$

where $K_h = \frac{\rho g K_p}{\mu}$ is the hydraulic conductivity of the porous matrix. Moreover, Forchheimer's nonlinear frictional term (denoted $\boldsymbol{R}_F$) is here neglected because of the low Reynolds number for this case. This term omitted, approximate solution for the difference in water level ΔH can be established. Numerical simulations were conducted with $K_h = 0.01\,m.s^{-1}$. Fig 2 shows a good agreement between the theoretical and numerical values of ΔH, as well as a smooth pressure field.

V. Conclusion

The present model correctly captures water infiltration in undeformable porous structure. A third test case with undeformable porous matrix is under achievement and will be presented during the workshop: the Polubarinova-Kochina schematic porous dam. The next step is to complete the model by taking into account the motion and deformation of the granular medium under water infiltration. To achieve this, the scheme presented at SPHERIC 2023 by De Sousa *et al.* [7] to model the structure will be used. First results will be shown at the conference.

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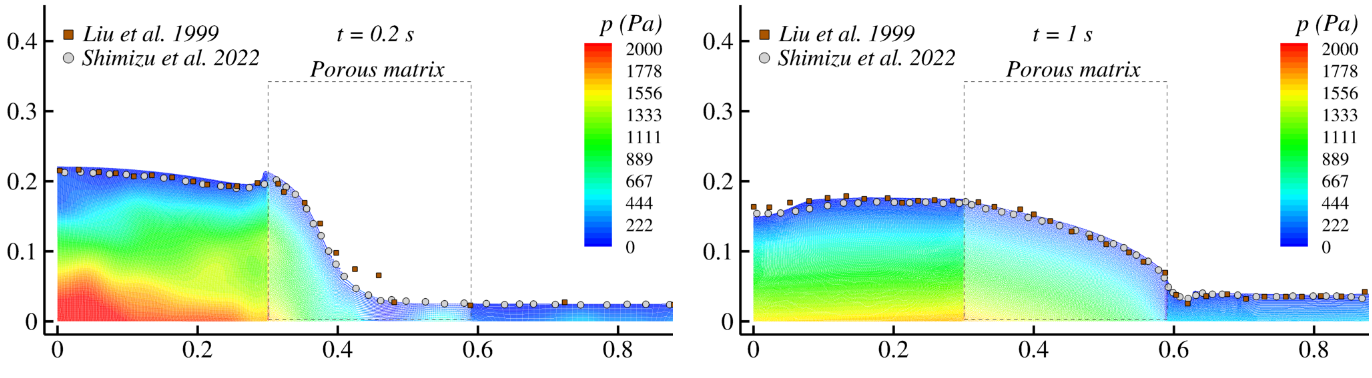
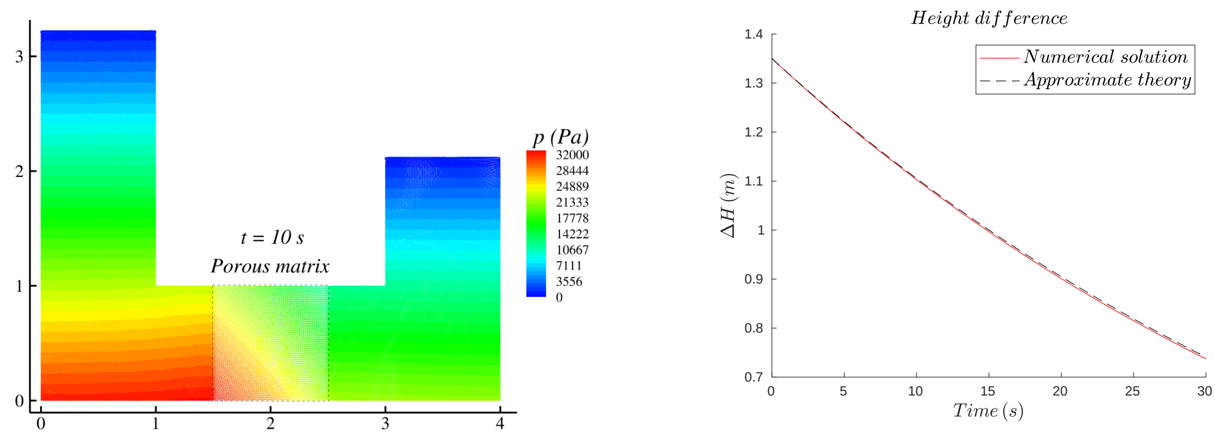


(a) (b)

Fig. 1. Dambreak case: Pressure field and comparison of the free surface with Shimizu *et al.* [4] and Liu *et al.* [11] at (a) $t = 0.2\,s$ and (b) $t = 1\,s$



(a) (b)

Fig. 2. U-tube case: (a) Pressure field at $t = 10\,s$ (b) Comparison of the height difference ΔH with approximate theoretical value over time

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