

Riemann based SPH with divergence cleaning

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I. INTRODUCTION

The Riemann based SPH scheme, originally proposed by Parshikov & Medin [1] is a variant of the classical weakly compressible SPH formulation, in which the stability is guaranteed by treating each particle – particle interaction as a Riemann problem. The absence of tuning parameters makes this approach attractive and potentially more robust than the classical weakly compressible formulation. In Riemann based SPH, a Riemann problem is solved in particle pairs in contrast to the average particle properties used in classical weakly compressible SPH. Nevertheless, the scheme is over dissipative in the absence of a dissipation limiter. Recently, low dissipation Riemann solvers in SPH have been proposed in the literature with the use of dissipation limiters and Monotonic Upstream-centred Scheme for Conservation Laws (MUSCL) reconstruction.

Although Riemann based SPH schemes are very effective on removing high frequency acoustic waves and provide accurate pressure fields, low frequency acoustic waves remain present and propagate through the domain with very little dissipation. This poses a significant challenge when simulating incompressible fluids by adopting a weakly compressible approximation in which the numerical speed of sound allows for 1% compressibility with impact flows. These acoustic waves, propagate through the domain and distort the pressure field. Recently, numerical remedies to filter the acoustic waves have emerged in (classical) weakly compressible SPH such the introduction of an additional diffusion term in the momentum equation [2] or the velocity-divergence error mitigating formulation [3].

Herein, in order to minimise the acoustic pressure waves arising from the weakly compressible assumption, we employ the highly effective divergence cleaning formulation of Fourtakas et al. [4] integrated in a Riemann-based SPH scheme. In this paper the constrained hyperbolic divergence cleaning is performed on the velocity field using the Riemann solution for the evolution of the ψ equation in space and time. The scheme is implemented in the open-source solver DualSPHysics [5]

II. NUMERICAL METHODS

Following the methodology of [1] a Riemann problem is constructed for each interpolating and neighbouring particle

pair i, j , respectively along their unit vector, within the support domain with an intermediate velocity and pressure,

$$u^* = \frac{\rho_L u_L + \rho_R u_R}{\rho_L + \rho_R} + \frac{P_L - P_R}{c_0(\rho_L + \rho_R)} \quad (1)$$

and

$$p^* = \frac{\rho_L P_R + \rho_R P_L}{\rho_L + \rho_R} + \beta \frac{\rho_L \rho_R (u_L - u_R)}{\rho_L + \rho_R} \quad (2)$$

where the subscript L and R denote left and right states, ρ is the density, u denotes the velocity, P is the pressure and c_0 the initial numerical speed of sound. The term β is a constrained velocity which is introduced to reduce the numerical dissipation at low Mach number [6].

A. Governing Equations

By projecting the intermediate velocity and pressure of (1) and (2) along the unit vector using a density weighted average of the physical and Riemann states properties, the discretised governing equations take the following form,

$$\frac{d\rho_i}{dt} = 2\rho_i \sum_j \frac{m_j}{\rho_j} (\mathbf{u}_i - \mathbf{U}^*) \nabla_i W_{ij} \quad (3)$$

$$\frac{d\mathbf{u}_i}{dt} = -2 \sum_j \frac{m_j}{\rho_j} (P^*) \nabla_i W_{ij} \quad (4)$$

using the usual SPH notation. Furthermore, in the presence of a wall boundary, the no-penetration and slip conditions are enforced by correcting the intermediate state by the hydrostatic condition and solving a partial Riemann problem.

Herein, viscous dissipation terms are discretised using the Morris operator using the physical particle properties. Furthermore, the Tait's equation of state has been employed with a 1% compressibility and the classical weakly compressible SPH assumptions remain valid [4].

Monotonic Upstream-centred Scheme for Conservation Laws (MUSCL) reconstruction is performed in accordance with,

$$\mathbf{u}_k = \mathbf{u}_i + \xi \left(\mathbf{x}_{ij} \cdot \frac{d\hat{\mathbf{u}}_i}{d\mathbf{x}_i} \right) \quad (5)$$

and

$$p_k = p_i + \varphi \left(\mathbf{x}_{ij} \cdot \frac{d\hat{p}_i}{d\mathbf{x}_i} \right) \quad (6)$$

where k denotes the Riemann states, ξ and φ are the van Leer limiter and the hat symbol indicates a corrected first order consistent SPH gradient [7].

B. Divergence cleaning using a Riemann formulation

We adopt the hyperbolic constrained divergence cleaning approach where a scalar field ψ is introduced in the momentum equation using a gradient term [4]. The ψ term evolves in time using a hyperbolic/parabolic equation,

$$\frac{D\psi}{Dt} = -c_0^2 \nabla \cdot \mathbf{u} - \frac{\psi}{\tau} \quad (7)$$

with τ a decay time scale,

$$\frac{1}{\tau} = \frac{\sigma c_i}{\lambda} \quad (8)$$

where typically $\sigma \in [0,1]$ is a free parameter and the characteristic length $\lambda = h$, defines how quickly the divergence errors decay, converging as $h \rightarrow 0$. The c_i term is the local particle speed of sound defined as,

$$c_i = \kappa \sqrt{\frac{p_i}{\rho_i}} \quad (9)$$

For further details on the hyperbolic/parabolic cleaning the reader is directed to Fournakos et al. [4].

The discrete form of (7) within the Riemann formalism takes the following form,

$$\frac{d\psi_i}{dt} = 2\rho_i c_0^2 \sum_j \frac{m_j}{\rho_j} (\mathbf{u}_i - \mathbf{U}^*) \nabla_i W_{ij} - \frac{\psi_i}{\tau} \quad (10)$$

which has a similar discrete form with the continuity equation (3). Note, that \mathbf{U}^* denotes the intermediate density weighted velocity of the Riemann problem and thus, (3), (4) and (10) which complete the weakly compressible solver are consistent.

III. RESULTS

Two impact flow test cases are presented on an unbounded and bounded domain to show the effectiveness of the proposed Riemann divergence cleaning weakly compressible scheme (R-div(u) SPH).

A. Patch test

The so-called patch test is a 2-D test case with two identical rectangular patches of liquid that impact at a 0° angle of incidence. Each patch length is $L = 2H$, with a height $H = 1$ m

centred around the impact point $x = [0,0]$, which will serve as the stagnation point. The patch velocity normal to the stagnation point is $u = 1$ m/s. Three different resolutions have been tested with $L/dx = [50,100,200]$ however, we will only discuss the middle resolution as the results do not vary significantly in terms of the velocity divergence errors. The Mach number is set at $Ma = 0.05$. Note that, the stability is guaranteed by the Riemann solver, and no viscosity or dissipation terms have been used in this case. The free parameter σ of (8) is taken as $\sigma = 1$.

Figure 1 shows a comparison of the pressure field at $t = 0.75$ s after impact for the R-div(u) SPH and classical SPH for $L/dx = 100$ where the effectiveness of the scheme to clean acoustic pressure waves using R-div(u) SPH is demonstrated.

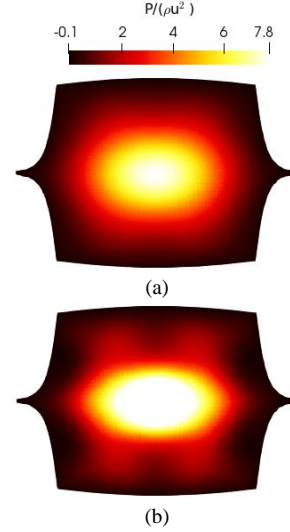


Figure 1. Pressure field at $t = 0.75$ s after impact for the (a) R-div(u) SPH and (b) classical SPH for $L/dx = 100$.

In Figure 2, the time history of the non-dimensional pressure considering both the R-div(u) SPH and the classical SPH is shown at the stagnation point with much more stable oscillations and a time decay trend as expected in this case.

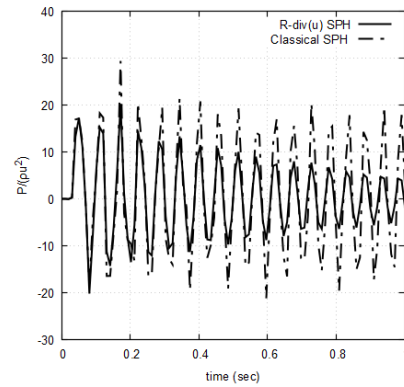


Figure 2. Time history of the pressure for $L/dx = 100$ and $Ma = 0.05$.

Figure 3 compares the divergence of velocity field at $t = 0.75$ s after impact using the R-div(u) SPH and classical SPH. The effectiveness of the cleaning algorithm is clearly demonstrated. Note, that we use a low dissipation Riemann formulation which does not provide any numerical dissipation for the long acoustic waves and any pressure wave filtering is solely performed by the R-div(u) SPH formulation.

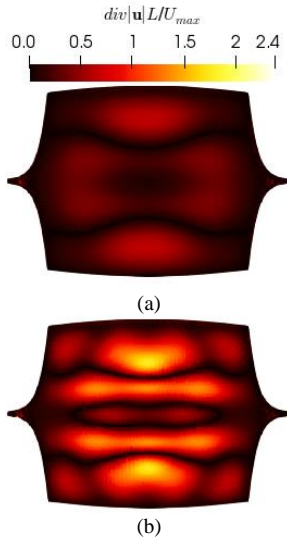


Figure 3. Divergence of velocity field at $t = 0.75$ s after impact for the (a) R-div(u) SPH and classical SPH for $L/dx = 100$.

B. Wedge impact on free surface

A symmetric free-falling wedge with no incline impacts a free surface with a velocity of $u = 6.15$ m/s. The impact in weakly compressible SPH generates a series of acoustic pressure waves as the wedge impacts the free surface.

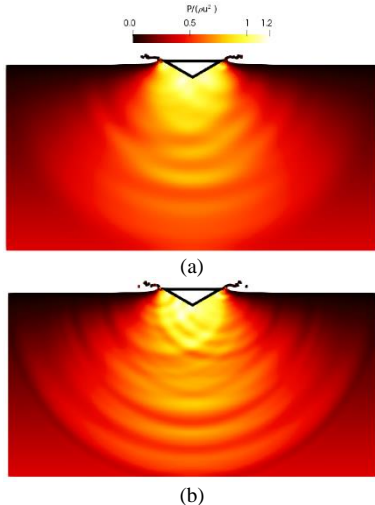


Figure 4. Pressure field at $t = 0.9$ s after the wedge impact for the (a) R-div(u) SPH and classical SPH for $L/dx = 376$.

A rectangular domain with $2L$ and height $L = 1.5$ m and open boundary conditions laterally is used with a wall boundary conditions at the bottom of the tank. Three different resolutions have been tested with $L/dx = [94, 188, 376]$. A Mach number of $Ma = 0.085$ is used to allow sufficient acoustic wave to travel the domain.

Figure 4 shows the pressure field at $t = 0.9$ s. At that instance, the acoustic pressure waves from the wedge impact have reached the bottom wall boundary. The effectiveness of the R-div(u) SPH formulation is evident not only on the pressure

plots of Figure 4 but also the divergence of velocity fields shown in Figure 5.

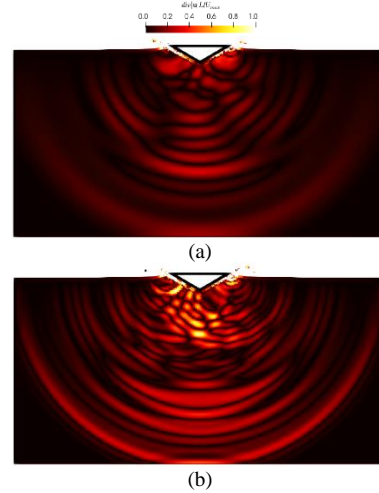


Figure 5. Divergence of velocity field at $t = 0.9$ s after the wedge impact for the (a) R-div(u) SPH and classical SPH for $L/dx = 376$.

IV. CONCLUSIONS

In the paper a novel Riemann based SPH divergence cleaning scheme has been presented. It has been shown that the proposed scheme is effective of removing the acoustic waves that arise from the compressibility assumption in weakly compressible SPH which is more prominent with low dissipation Riemann solver formulations.

ACKNOWLEDGMENT

The authors would like to acknowledge Francesco Ricci for their contribution providing post-processing scripts of the results presented in the wedge impact case.

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