AN ENHANCED INLET BOUNDARY CONDITION FOR STABLE AND ACCURATE NONLINEAR WAVE SIMULATIONS

Taiga Kanehira, Samuel Draycott, Benedict Rogers, Peter Stansby
Department of Civil Engineering and Management
University of Manchester, Manchester, UK
taiga.kanehira@manchester.ac.uk

I. INTRODUCTION

Three-dimensional effects play a crucial role in the dynamics of ocean waves affecting wave growth and limiting forms. A recent study [1] revealed that directionally spread ocean waves can grow up to twice as large as previously estimated under conventional unidirectional sea states. Therefore, further investigation into the three-dimensional modelling of waves, including the associated breaking phenomena, is essential for the design of offshore structures as well as understanding CO_2 absorption and the transport of microplastics in the ocean.

Owing to their robustness, particle methods have been utilized for modelling freak waves and Fluid and Structure Interactions (FSI). However, challenges persist when extending these methods to three-dimensional waves. For example, while numerous modelling approaches have been developed [2, 3 and 4], their applicability remains largely confined to conventional waves in a two-dimensional vertical plane domain. We aim to improve open Boundary Conditions (BCs) [2] in a three-dimensional domain as depicted in Fig. 1.

In [2], Tafuni et al. introduce an open boundary condition for weekly compressible Smoothed Particle Hydrodynamics (WCSPH), enabling inflow and outflow BCs in the well-known open-source code, DualSPHysics [5]. Based on the shallow water wave theory, Verbrugghe et al. [3] implemented a velocity correction into [2] and applied the method for FSI problems within one wavelength. Tsuruta el al. [4] proposed the wavy interface model for wave generation in the Moving Particle Semi-implicit method (MPS) [6]. This method uses only time series data of target wave elevation at an interface, avoiding troublesome incident wave tuning. However, these methods require a particle generation and removal algorithms for the free surface, complicating their extension to 3D ocean wave problems. Modelling wave paddles [7] offers an alternative for reproduction of 3D wave fields but necessitates a large computational domain and high cost.

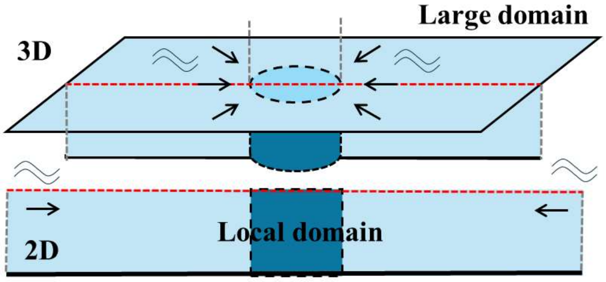

Figure 1. Concept of coupling model in a 3D wave field.

The present paper is organized as follows: Section 2 describes the numerical method and proposed BCs. Based on [3], second order Stokes waves are generated for numerical validation. We will discuss wave dynamics simulation as well as numerical accuracy in Section 3. Finally, Section 4 concludes the paper by summarizing the key outcomes and highlighting potential directions for future research and practical applications.

II. NUMERICAL METHOD

A. *Smoothed Particle Hydrodynamics Model*

For the localized simulation shown in Fig. 1, the open-source Lagrangian solver DualSPHysics v5.2 [5] is used. The solver, based on the SPH formulation, governs fluid motion with the Navier-Stokes equations and efficiently handles complex free-surface flow with large deformations. Computational speed is enhanced through GPU-based parallel computing and explicit pressure calculations using the equation of state. Therefore, it holds potential for future extension to three-dimensional simulation, as depicted in Fig. 1.

B. *Modification for the Open Boundary Condition [2]*

In [2] (referred to as Inlet U here), horizontal velocity and water elevations are assigned to buffer particles, involving particle creation and removal based on the water elevation. However, extending this to a 3D region requires complex coding, and for general waves, particularly under deep water conditions, vertical motion cannot be neglected. Moreover, this paper is the first to reveal significant disturbances in the wave field near the connection interface of the boundaries.

In this study, we propose a new method (Inlet UW) that assigns both horizontal and vertical velocities to Inlet particles. By allowing fluid motion in both horizontal and vertical directions within 2D plane, this method enables direct tracking of water elevations without relying on particle creation or removal. This method enables the direct application of velocity conditions that reflect the orbital motion of fluid particles, ensuring physical consistency between the boundary conditions and the fluid motion governed by the Navier–Stokes equations. Additionally, by incorporating vertical velocity components, the method helps to prevent the underestimation of kinetic energy associated with orbital motion.

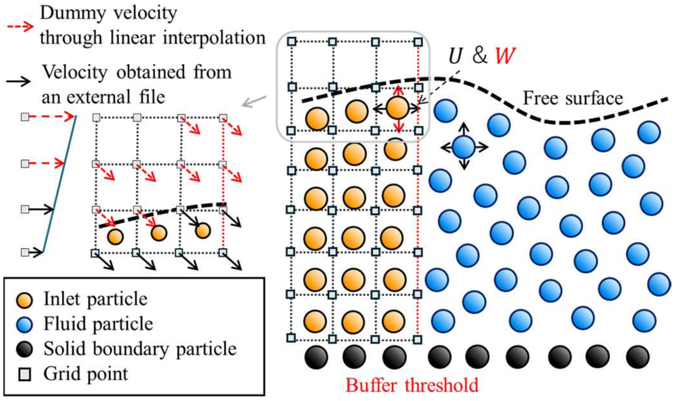

Figure 2. Schematic image of the proposed Inlet BC.

Fig. 2 illustrates the concept of the proposed Inlet boundary conditions. Horizontal and vertical velocities are bilinearly interpolated from external grid-based velocity data (obtained from OceanWave3D [8], referred to as OW3D hereinafter) and assigned to the Inlet particles. To address the undefined velocities at grid points above the free surface, dummy velocities (represented by red dashed vectors) are introduced. These dummy velocities prevent underestimation of velocity values near the free surface during bilinear interpolation, suppressing detachment of Inlet particles from the water surface. The density of the Inlet particles is mirrored from the fluid domain as implemented in [2]. Additionally, due to the difficulty of generalization, no new Inlet particles are generated at the left side of the buffer zone when Inlet particles penetrate the fluid domain.

C. *Wave Propagation Test*

To evaluate the applicability and accuracy of the Inlet UW BCs, second-order Stokes waves were generated in a wave tank (Fig. 3) under intermediate and deep-water conditions based on [3]. Results were compared for water surface elevation, velocity fields, and Stokes drift (a net horizontal drift in the wave direction). As shown in Fig. 1, this method targets a localized region of one to two wave lengths (L), so wave gauges were placed within two wave lengths near the wave generation boundary. The particle distance d_p was 0.01 m and the wave simulation period was 100 s. For the intermediate depth conditions ($ak = 0.05, kh = 1.2$) depicted in Fig. 3, wave height (H) is 0.08 m, wave period (T) is 2 s, wavelength (L) is 5.22 m, and water depth (h) is 1.0 m. Three wave generation methods were tested: Inlet U, InOut U+ (InOut U with velocity correction based on shallow water theory in [3]), and the proposed Inlet UW. Note that both Inlet and Outlet condition ware used in InOut U+ based on [3], which is why it is named "InOut" here.

The Stokes drift (D_{SD}) may be expressed as follows:

$$D_{SD} = \int_{t_1}^{t_2} u_{SD} dt \tag{1}$$

$$u_{SD} = c(ak)^2 \cosh\big(2k(h+z)\big)/(2\sinh^2(kh)) \tag{2}$$

where u_{SD} represents the Stokes drift velocity for the general water depth h in [9], c is the phase speed, $a = H/2$ denotes the wave amplitude, and k is the wave number. The times t_1 and t_2 are arbitrary. However, considering that Stokes drift is related to non-closed orbital motion of the non-linear waves, it seems reasonable to select times after the wave field has sufficiently developed. In this study, t_1 is defined as the time when the well-developed wave reaches an arbitrary x-coordinate, and t_2 as the end time of the simulation.

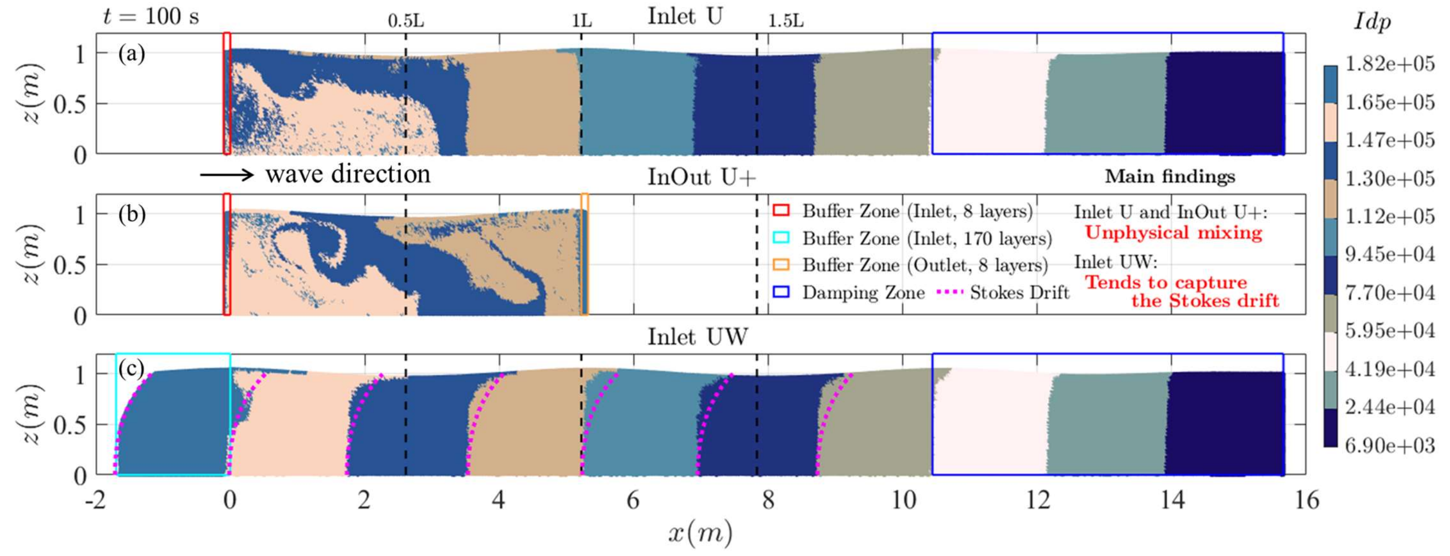

Figure 3. Stokes drift motion represented in the simulated wave field

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Simulated Wave field and Numerical Validation

Fig. 3 shows the simulated second-order Stokes waves in intermediate-depth water. The red rectangle indicates the Inlet BCs for wave generation and blue damping zone using velocity decay. Water surface elevations η were measured at x = 0.5, 1.0 and 1.5 L, with comparison normalized by the target wave amplitude a shown in Fig.4. Inlet UW and InOut U+ exhibit pretty good agreement, while, a slight phase shift is observed in Inlet U, likely due to unphysical counterclockwise circulation near the interface shown in Fig. 3a. Comparing the elevations at the early stage of the simulation (Figs. 4a, and 4c) with the end (Figs. 4b and 4d), the elevations simulated using the Inlet UW exhibit a slight water setup, likely due to the Stokes drift coupled with a semi-enclosed domain. This drift increases the total fluid volume in the tank, as no outflow occurs at the tank's end with the damping zone. Although the InOut U+ shows better agreement in the comparison of the free surface (Figs 4a and 4b), attributed to the applied velocity correction, minor fluctuations are evident in the inset comparisons (blue dash-dotted line). This is likely a result of unphysical circulation as shown in Fig. 3b.

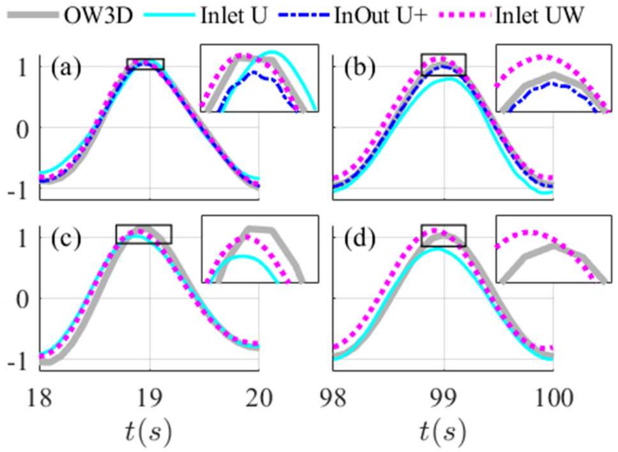

Figure 4. Comparison of the water surface elevations at x = 0.5 L (a, b) and 1.5 L (c, d) near the start (a, c) and end (b, d) of the simulations

B. Stokes Drift

Although the time series of water surface elevation appears stable, Inlet U and InOut U+ exhibit unphysical and unstable fluid motion. Fig. 3 shows the net particle displacement using colour shading, with the colour scale indicating particle indices (Idp). The top two panels (Inlet U and InOut U+) display fluid instabilities, while the bottom panel (Inlet UW) demonstrates smoother and more physically consistent particle distribution. The analytical Stokes drift, calculated using (1), is represented by the magenta dotted line in Fig.3c. The simulated drift in the buffer zone exhibits good agreement, whereas the particle drift in the fluid domain is underestimated. As the simulation progresses, we observed that the underestimation of Stokes drift becomes more pronounced.

This underestimation can primarily be attributed to two factors. First, the no-outflow condition at the tank's end causes the accumulation of inflow driven by Stokes drift, altering the orbital motion of fluid particles. Second, wave decay along the x-direction results in energy loss, which contributes to the underestimation. Since Stokes drift is proportional to the wave steepness (ak) as shown in (2), the reduction in wave height (a) further exacerbates this issue. However, in comparison with the results in Figs 3a and 3b, our proposed method (Fig. 3c) captures the trend well, demonstrating the advantage of the Lagrangian approach in SPH.

IV. CONCLUSIONS

To facilitate coupling in a three-dimensional domain, we proposed modified open boundary conditions (Inlet UW) for wave modelling. In addition to achieving high accuracy in time-series comparisons of water surface elevation, the method enables appropriate reproduction of wave dynamics such as Stokes drift and improves computational stability. Future work will focus on extending the proposed method to fully three-dimensional wave fields and exploring its applicability to complex fluid-structure interaction problems and simulation of three-dimensional freak waves.

ACKNOWLEDGMENT

This work was supported by the Japan Society for the Promotion of Science (JSPS) Overseas Research Fellowships.

References

[1] M. L. McAllister, S. Draycott, R. Calvert et al., “Three-dimensional wave breaking”, Nature 633, 2024, pp. 601–607.

[2] A. Tafuni, J. M. Domínguez, R. Vacondio, and A. J. C. Crespo, “A versatile algorithm for the treatment of open boundary conditions in Smoothed Particle Hydrodynamics GPU models”, Comput. Methods Appl. Mech. Eng., vol. 342, 2018, pp. 604–624.

[3] T. Verbrugghe, J.M. Domínguez, C. Altomare et al., “Non-linear wave generation and absorption using open boundaries within DualSPHysics”, Computer Physics Communications, vol. 240, 2019, pp. 46–59.

[4] N. Tsuruta, A. Khayyer, H. Gotoh, and K. Suzuki, “Development of Wavy Interface model for wave generation by the projection-based particle methods”, Coastal Eng., vol. 165, 2021, 103861.

[5] J. M. Domínguez, G. Fourtakas, C. Altomare et al., “DualSPHysics: from fluid dynamics to multiphysics problems”, Comp. Part. Mech. 9, 2022, pp.867–895.

[6] S. Koshizuka, and Y. Oka, “Moving-Particle Semi-Implicit Method for Fragmentation of Incompressible Fluid”, Nuclear Science and Engineering, 123(3), 1996, pp. 421–434.

[7] T. Kanehira, M. L. McAllister, S. Draycott et al., “Highly directionally spread, overturning breaking waves modelled with Smoothed Particle Hydrodynamics: A case study involving the Draupner wave”, Ocean Model., vol. 164, 2021, 101822.

[8] A. P. Engsig-Karup, H. B. Bingham and O. Lindberg, “An efficient flexible-order model for 3D nonlinear water waves”, J. Comput. Phys., 228, 2009, pp. 2100–2118.

[9] F. Ursell, “The long-wave paradox in the theory of gravity waves”, Math. Proc. Cambridge Phil. Soc. 49, 1953, pp. 685–694.