THE PRESSURE SIGNAL IN SIMULATIONS OF WEAKLY COMPRESSIBLE SPH

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I. Introduction

SPH simulations of artificially weakly compressible flows resolve density and pressure fields as coupled by the equation of state. Changes of those fields travel across the fluid at the constant speed of sound defined by the modeller. Besides, the time series of density/pressure at locations of choice can be recorded at each time level during a simulation. If the discrete Fourier transform (DFT) of these signals displays recognisable behaviours in certain ranges, the spectra appear to tell something about meaningful occurrences in the simulations. As recognised in [1], the harmonic components decay at different rates and with excursions of varying amplitude within frequency bands of varying width. The limits of these bands seem to be associated with key discretization parameters of SPH like the particle spacing, the kernel support size and the artificial speed of sound.

This contribution explores further the information present in the discrete spectra of the density/pressure measured in weakly compressible SPH simulations, as well as their possible uses in the modelling practice. The ongoing investigations proceed inductively by means of numerical experiments and elemental mathematical interpretations. Selected descriptive results are presented in outline hereinafter. As expected, the harmonic components reflect the large-scale response to external forcing or to imbalance in the initial conditions. Likewise, their distribution may indicate how modelling choices in SPH and their (side) effects affect the harmonic components at appropriate frequency scales, say, flavours of the governing equations, numerical parameters, spurious events. Reading the density/pressure signals can thus be an effective diagnostic tool for the modellers to monitor the simulation development and crosscheck their expectations, even during runtime. Its advantages may stand out as the simulation size grows into millions of particles and time steps.

II. Material and methods

The signal described next is the pressure measured at a wall hit by the water released on a dry floor from a dam break. The simulations are the 2D reduction, in dimensionless form with Ma $= 0.1$, of experiments at Re $= 3.8$ - $5.5 \cdot 10^6$ [2]. The solver is DualSPHysics [3].[1] The highly resolved simulations previously deposited in an openly accessible repository serve as baseline [4].[2] The code adaptations and input settings are detailed in [1] and only a selection is recalled hereinafter for context. A density diffusion term (DDT) is applied [5], and particle shifting is disabled [6]. The density only depends on pressure, and the equation of state is linear. The maximum resolution of the water column attained, $d/\Delta x = 6400$, corresponds to $\text{Re}_{\text{eff}} = 256,000$ based on the Reynolds scaling of [7], to about 82M particles and to about 45 days runtime. The wall pressure is computed as the average normal force over as wide a line as the kernel support diameter, like with a numerical hydrophone. The pressures computed in the baseline do converge towards the experimental measurement as the particle-number density increases eightfold. Importantly, based on an approximate evaluation of the boundary-layer thickness, the viscous sublayer gets resolved upwards of $\text{Re}_{\text{eff}} \approx 53,000$. The wall pressure signals at high particle-number density therefore capture information on a wide range of scales, from the impact of the water bulk down to laminar motions. The results commented here are obtained for $d/\Delta x = 3200$ ($\text{Re}_{\text{eff}} = 128,000$; 20M particles).

The absence of air and surface tension are the main modelling limitations, due to the usual trade-off of compute resources. Such limitations affect a dam-break simulation severely only after the water rising up the wall bounces back on the incoming flow. Although unrealistic, the ensuing shedding of droplets, appearance of voids and pervasive whirling are a blessing in disguise for the spectral analysis, however. Those chaotic outcomes reproduce known shortcomings of SPH, and provide the playground to investigate the activity and removal of undesired phenomena.

III. Results and discussion

The scaling units of the dimensionless handling are omitted for simplicity.[3]

[1]DualSPHysics is free software distributed under a LGPL licence. In fulfilment of the licensing terms, this study implies neither endorsement nor promotion of DualSPHysics, and the citation of DualSPHysics implies neither endorsement nor promotion of this study from the DualSPHysics contributors. This study should not be construed as a validation exercise of DualSPHysics.

[2]The supplementary animations at https://www.youtube.com/playlist?list=PLb_klyJ6w5QihDlztSqN0GRhT7awNnibe show the fields of the available quantities of interest. The pressure signals discussed here are not part of [4].

[3] $\sqrt{gd}$ for speed, $\sqrt{g/d}$ for frequency, $\sqrt{d/g}$ for time, gd for pressure.

A. Results

The upper Fig. 1 shows the full pressure signal and the division of the water motion into four stages with different dominant dynamics, as detailed in the caption. The corresponding spectrum in the lower panel also marks with vertical lines two key frequency scales. The slower one, $c_0/4h$, relates to the kernel diameter and here the probe size, and identifies the threshold between low and high frequencies (LF, HF). The fastest scale, $c_0/(Ch)$, is the frequency beyond which oscillations are permitted by viscous interactions. (C is the Courant coefficient.) Those very high frequencies (VHF) can indeed exist because the stability criterion determines the smallest time step in the simulation, and its highest frequency, according to:

$$\Delta t_{\text{stability}} = C \min_i \frac{1}{\frac{c_0}{h} + \max_j \frac{|\mathbf{u}_{ij} \cdot \mathbf{r}_{ij}|}{\mathbf{r}_{ij}^2}} < C \frac{h}{c_0}$$

where the second argument in the denominator is the same functional as the Laplacian operator. Finally, the grey lines provide qualitative contrast by showing a simulation with fewer particles and without density diffusion term, as detailed in the caption — commenting on this is left for the conference proper.

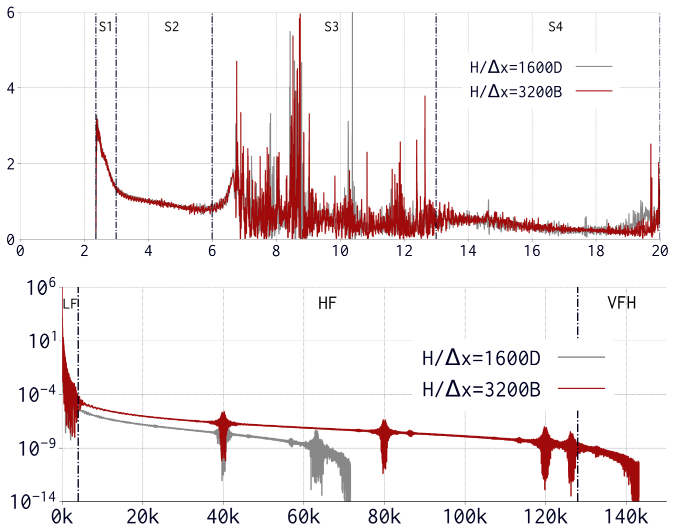

Fig. 1. *Top*: Pressure signal in the plane time-pressure. The vertical lines delimit the flow stages of S1) surge impact and rise; S2) fall and rebound; S3) collapse, spraying and cavitation; S4) sloshing and settling. See footnote 2 for animations of the flow fields. *Bottom*: Pressure spectrum of the full signal in the plane frequency-spectral energy. The vertical lines separate low, high and very high frequency bands (LF, HF, VHF). *Both*: Red lines are the baseline results (label 3200B). Grey lines display results for contrast, without density diffusion term and with $d/\Delta x = 1600$, $\text{Re}_{\text{eff}} = 56,000$, 5M particles (label 1600D).

The spectrum of Fig. 1 encodes information from the time-domain signal in its entirety. Fig. 2 instead shows the pressure spectra computed for the four stages separately. The snapshots of flow fields shown for illustration on the right display the tank corner close to the pressure probe; the signals gather information propagating across the entire domain, however [1].

The focus here lies on the wide HF and VHF bands, which reflect the small time steps typical of SPH. While contributing negligibly to the overall signal, this frequency range still relays insights in the actual dynamics simulated in each stage.

For example, the oscillations in the VHF band vanish only in stage 1, that is, during the forward impact on the wall; the same behaviour does not occur with the subsequent onset of recirculating vortexes and of chaotic agitation. Then, a feature of stage 3 is the concentration of spectral excursions in narrow bands ('tones') precisely at the frequencies of 40k, 80k and 120k $\sqrt{g/d}$. The snapshots in Fig. 2 and the animations show that dominant and unique features of stage 3 are the spurious fragmentation into droplets, showering the fluid as result, and the generation and destruction of spurious voids. The intriguing regularity in the tones frequency is to be explored further.

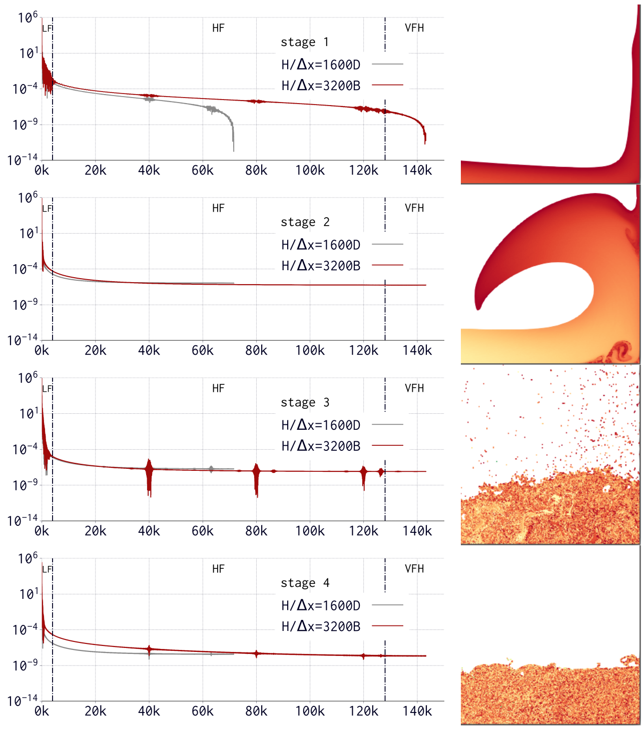

Fig. 2. *Left*: Pressure spectra in the four flow stages in top-down time order. See the caption of Fig. 1. *Right*: Flow fields at the tank wall at the end time of each stage to the left. Colour shades indicate immutable markers for the initial particle position and express the effectiveness of mixing-inducing flow patterns.

Finally, the less resolved simulations differ mostly in the LF band and in the decay rates in some stages. Numerical experiments, left to the presentation, disentangle the effects of the particle-number density and of the density diffusion term.

B. Discussion

Using pressure signals to investigate simulations calls for the consideration of several aspects. Firstly, the approach comes with its own parameters and alternatives. For example, here the pressure has been recorded on a sensing area at the wall via the momentum equations (the numerical hydrophone), whereas it can also be recovered in the inner fluid via density and the equation of state. The size of the sensing area arguably affects the separation in frequency bands, although lower resolutions may mask that effect [1]. That SPH simulations typically march at a variable time step is also relevant for handling spectra. Also, since certain bands gain salience in different stages of the flow, the signal could also be interpreted using spectrograms. Additionally, signals should be sampled in other locations and for other flow situations so as to confirm the reliability and validity of this approach more broadly. Finally, more questions regard whether each modelling choice impacts just a single band or the entire frequency range; or to which extent the large and small scales are disconnected in weakly compressible flows.

IV. Conclusion

Setting up a simulation entails decisions on parameters aplenty defining the physics to model, the algorithms to follow and the codes to compile and run [8]. Increasing the particle-number density or the simulated time makes the descriptions of flow fields voluminous, especially due to the stability constraint in the time stepping. Pressure signals seem to have some nice properties [9] to assist the workflow of SPH modellers.

Consistency. We talk interchangeably of density and pressure thanks to the working assumption of weak compressibility. Weak compressibility makes the attending partial differential equations of hyperbolic-parabolic type, whereby simulations resolve the propagation of density waves in finite times. The constant sound of speed also locks the frequencies and wavelengths of temporal and spatial oscillations and ensures that such waves are free from dispersion. The compression and expansion of particles is thus a vector of information about flow-following patterns, waves and impulses, realistic and spurious effects alike. Thanks to this fundamental consistence, the signals can also return interpretable and meaningful information on a numerical solution.

Exactitude. The signal and spectrum record information down to the smallest scales computed, save Nyquist's theorem. Spectral analysis can probe simulations with absolute sensitivity insofar as the signals capture all events that have been calculated and thus occurred in the shape of crunched numbers. However, spectral analysis is not an absolutely specific test, owing to flow events devoid of acoustic traces. For a start, the evolution of vorticity is physically unrelated to pressure, and the harmonic components of density/pressure arguably relay the development of irrotational features only. Therefore, pressure signals cannot replace the inspection of the simulated flow fields completely.

Lightness. The arrays of a time series and a spectral transform require considerably less storage than equally spaced sequences of domain-wide flow fields. The spectra are thus a compact representation of what happened in a flow simulation.

Quickness. The modeller can rapidly collect density/pressure signals at runtime, and the numerical recipes for DFTs are highly optimised for speed, with well understood properties and pitfalls. Spectral analysis of density/pressure signals is therefore a computationally efficient diagnostic tool to capture on the fly what is happening in the fluid domain during a simulation.

Visibility. Visual inspection of domain-wide flow fields fails at high enough particle count, either because fast oscillations entail displacements smaller than the pixel size, or because the frame rate is in effect a low-pass filter. As the simulation size scales up, the task of inspecting flow fields at narrowly spaced intervals becomes cognitively overwhelming. So, indirect sound signals are more tangible a representation of the dynamic range in the motion than direct visual rendering. A spectrum can be more expressive of the flow dynamics than detailed flow fields.

Multiplicity of use. Once theoretical or heuristic expectations give to the effects of modelling choices a place in the spectrum, from the spectral content we can then read how the simulation responded to our interventions. Spectra or spectrograms could be used to explore, tune and compare the large search space of modelling options. The analysis of pressure signals could then help both in the conceptualisation of work hypotheses as well as in their software implementation. This approach can possibly work for other simulation systems resolving wave propagation.

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