Compared SPH and LBM approaches for FSI of trans-aortic flows

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I. INTRODUCTION

Cardiovascular diseases has become one of the predominant cause of death in developed countries, and valve failures of elderly patients represents a large percentage of such incidents. For those patients, the Trans-Aortic Valve Implantation (TAVI) procedure where a bio-prosthetic valve replaces the native valve, has become standard. In ambiguous cases, the choice of the valve's dimension relies on the surgeon's experience.

Regarding Fluid Structure Interaction (FSI) literature of a pulsatile blood flow through a deformable tricuspid aortic valve, Li *et al.* [7] developed and validated a partitioned strong coupling between Lattice Boltzmann Method (LBM) and Finite Element Method (FEM) frameworks. Similarly for Smoothed Particles Hydrodynamics (SPH) method, Mao *et al.* [8] coupled the SPH method with the FEM using the Abaqus commercial software and obtained good results in comparison with experimental data although their numerical aorta model was simplified. More recently, using an SPH only framework, Laha *et al.* [6] were able to simulate FSI for trans-aortic flow with both mechanical and bio-prosthetic valves. The comparison of biomarkers such as Time Averaged Wall Shear Stress (TAWSS) and Effective Orifice Area (EOA) for the two valve types shows an increased TAWSS for the mechanical one.

In this overall context, this work focuses on the comparison of performance and precision between the SPH-FEM and the LBM-FEM coupling frameworks. Although the LBM is found easy to implement and provides fast computation, the Lagrangian behavior of the SPH method makes it suitable for robust and accurate simulations of Fluid-Structure Interactions. Indeed, Eulerian based methods suffers from numerical issues caused by mesh distortions with large solid deformations, while the use of Immersed Boundary Method (IBM) as in LBM leads to the inability to capture accurately local stress fields. The final aim is to develop and choose the best numerical tool to provide deeper insights for the surgeons and therefore to improve the TAVI procedure.

II. NUMERICAL TOOLS

A. Fluid formulations

In this work, two fluid formulations have been used with different strengths and weaknesses that will be discussed in the following section for the specific case of a pulsatile flow through a deformable tricuspid aortic valve. This subsection provides a brief description of both formulations.

Lattice-Boltzmann Method (LBM):

In the present work, the LBM is based on the discretised Lattice Boltzmann equation for isothermal weakly-compressible single-phase fluid flow, as derived by Guo *et al.* [4] from the continuous Boltzmann equation using the Bhatnagar-Gross-Krook (BGK) collision model. This model reads:

$$\begin{aligned} f_\alpha(\mathbf{x}+\boldsymbol{\xi}_\alpha\Delta t,\quad t+\Delta t) = f_\alpha(\mathbf{x},t) - \tfrac{\Delta t}{\tau}\left(f_\alpha(\mathbf{x},t) - \right. \\ \left. f_\alpha^{eq}(\mathbf{x},t)\right) + \Delta t\left(1-\tfrac{\Delta t}{2\tau}\right)\mathcal{F}_\alpha(\mathbf{x},t), \end{aligned} \tag{1}$$

with $\boldsymbol{\xi}$ the discrete velocity direction, $f_\alpha(\mathbf{x},t)$ the distribution function along the αth lattice direction, $\mathcal{F}_\alpha(\mathbf{x},t)$ a body-force term, Δt the discrete time-step, τ the relaxation time factor from $\nu_f = c_f^2(\tau - 0.5\Delta t)$ with ν_f the kinematic viscosity. The macroscopic fluid density ρ_f and velocity $\mathbf{v}_f$ are computed as

$$\rho_f = \sum_\alpha f_\alpha, \quad \rho_f\mathbf{v}_f = \sum_\alpha \boldsymbol{\xi}_\alpha f_\alpha + \frac{\Delta t}{2}\rho_f\mathbf{g}\ , \tag{2}$$

and the macroscopic pressure p is calculated using the barotropic equation of state $p = \rho_f c_f^2$. Finally, a stabilization technique based on the Hermite polynomial expansion [12] is retained to enhance the numerical stability of the adopted LBM formulation.

Smoothed Particle Hydrodynamics (SPH):

Concerning the SPH formulation, several choices have been made in order to accurately simulate the pulsatile flow through the valve. Firstly, this case suffers from Tensile Instability (TI) and therefore requires a specific treatment. In this study, accurate but non-conservative pressure gradient [10] is used to prevent this issue and obtain accurate results.

A second important point concerns particle disordering. Employing a Particle Shifting Technique (PST) is essential to break the particle in-line structures alongside the Lagrangian trajectories that highly deteriorates the numerical results. For that purpose, the PST velocity derived in [9] is employed. Note that the quasi-Lagrangian derivative $\frac{d(\bullet)}{dt} := \frac{\partial(\bullet)}{\partial t} + \nabla(\bullet)\cdot(\boldsymbol{v}+\boldsymbol{\delta v})$, is used. The SPH formulation finally reads :

$$\begin{cases} \dfrac{\mathrm{d}\rho_i}{\mathrm{d}t} & = -\rho_i \langle div\ \boldsymbol{v} \rangle_i^- \ - \ \rho_i \langle div\ \boldsymbol{\delta v} \rangle_i^- \\ & \quad + \langle div\ \rho \boldsymbol{\delta v} \rangle_i^+ \ + \ \Theta^{\rho}_{i,Rie} \\ \rho_i \dfrac{\mathrm{d}\boldsymbol{v}_i}{\mathrm{d}t} & = -\langle \boldsymbol{grad}\ P \rangle_i^- \ + \ \boldsymbol{F}_i^{\mu} \ + \ \langle div\ (\rho \boldsymbol{v} \otimes \boldsymbol{\delta v}) \rangle_i^+ \\ & \quad - \boldsymbol{v}_i \langle div\ \rho \boldsymbol{\delta v}) \rangle_i^+ \ + \ \boldsymbol{\Theta}^{\boldsymbol{v}}_{i,Rie} \\ \dfrac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}t} & = \boldsymbol{v}_i + \boldsymbol{\delta v}_i, \quad V_i(t) = m_i \,/ \rho_i(t), \\ & \quad P_i = c_0^2(\rho_i - \rho_0) \end{cases} \tag{3}$$

where $\Theta^{\rho}_{i,Rie}$ and $\boldsymbol{\Theta}^{\boldsymbol{v}}_{i,Rie}$ denote the numerical diffusive terms obtained by means of Riemann solver.

To handle the complex geometries present in the targeted application, a Boundary Integral Method (BIM) is used [2], [3]. A Strong Stability Preserving Runge-Kutta scheme (SSPRK4-3) is finally employed for the time integration of the scheme (3).

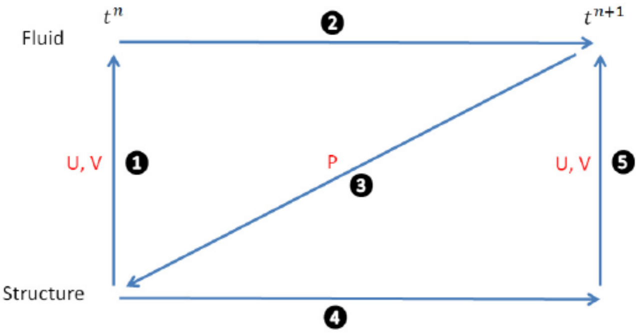

Fig. 1. Conventional Serial Staggered (CSS) scheme

B. Solid model : Finite Element Method (FEM)

The FEM framework is based on the total Lagrangian formulation considering non-linear displacements and strains hypotheses [1]. The momentum equation is formulated as

$$\boldsymbol{\nabla}_0 \cdot \mathbf{P} + \rho_0 \mathbf{b} = \rho_0 \mathbf{a}_s \ , \tag{4}$$

with $\rho_0 = \rho_0(\mathbf{X}, t)$ the initial solid density, $\mathbf{b}$ the body-force, $\mathbf{a}_s$ the solid acceleration, $\boldsymbol{\nabla}_0 \cdot$ the divergence operator with respect to $\mathbf{X}$ and $\mathbf{P} = \mathbf{P}(\mathbf{X}, t)$ being the nominal (1^{st} Piola-Kirchhoff) stress tensor where $\mathbf{X}$ are material coordinates and t is the time. The hyperelastic Neo-Hookean material model retained from [1] reads

$$\mathbf{S} = 2\frac{\partial \Psi}{\partial \mathbf{C}} = \lambda_0 (\ln J) \mathbf{C}^{-1} + \mu_0 (\mathbf{I} - \mathbf{C}^{-1}) \tag{5}$$

with λ_0 and μ_0 the first and second Lamé coefficients respectively and $\mathbf{C} = \mathbf{F}^T \mathbf{F}$ is the Right Cauchy-Green strain tensor.

Time integration is performed using the predictor-corrector Newmark implicit time scheme and stabilization is obtained via the generalized-α method [5].

C. Fluid-Structure Interactions (FSI)

The SPH-FEM coupling framework is based on a partitioned approach with a Conventional Serial Staggered (CSS) scheme in which the SPH solver explicitly solves Equations (3) considering a solid state at the current instant to compute the external force F^{n+1}_{ext} sent to the FEM solver, which implicitly updates the solid state the next instant as represented in Fig. 1.

III. RESULTS AND DISCUSSIONS

A comparison is carried out between both numerical tools developed for the present study, *i.e.* (i) Weak coupling between LBM and FEM with implicit IBM and an implicit time integrator for the deformable structure and (ii) Weak coupling between SPH and FEM with an explicit time integration for fluid and an implicit one for solid. Such comparison is entirely performed with the simulation of a 3D blood flow through a modeled aortic valve bio-prosthesis loaded by a pulsatile inlet and set in a simplified aorta geometry with Vasalva sinuses. A detailed set-up of the test-case is available in [7]. This investigation is generally based around three main elements: a comparison of velocity fields, a study about biomarkers and an evaluation of the CPU costs.

For the fluid spatial resolution, a ratio $\frac{D}{\Delta x} = 500$ is used for the LBM simulation against $\frac{D}{\Delta x} = 417$ for SPH, with $D = 0.25$ the inlet part diameter. Regarding time-steps, the LBM is limited to $\Delta t_{LBM} = 5.10^{-5}$ s while for the chosen SPH time-integrator, a CFL number 0.4 and a speed of sound $c_0 = 8.0$ m.s^{-1} allows for $\Delta t_{SPH} = 9.10^{-5} \approx 1.8 \times \Delta t_{LBM}$. Firstly, a qualitative overview of the results for both methods is provided. At peak systole ($t = 0.26s$), the axial velocity field visualized in a sagittal plane, defined by xz-plane at $y = 0.022m$, is overestimated for SPH, likely due to a smaller opening of the valve, see Fig. 2. Note that SPH formulation is not affected by Tensile Instability thanks to the non-conservative pressure gradients, and regular fields are observed even near the complex geometry. An underestimation of the forces involved in the weak coupling however seems to be responsible for the smaller opening motion.

A comparison of the valve opening is provided in Fig. 3, displaying the Effective Orifice Area over time. Results from Li *et al.* [7] for a coupling between LBM and FEM numerical methods with an implicit Immersed Boundary Method and an explicit integration for the solid structure, which has already been validated for this case against experimental and numerical results [11], is added as a reference solution. Good agreement is found between FSI solutions provided by LBM-FEM formulations, while a clear underestimation of the EOA is visible for the SPH-FEM formulation.

Finally, the present SPH-FEM results have been obtained using 70 processors for the fluid domain with a computational time of 16.75 h for 0.82 s of physical time. In comparison, the LBM-FEM framework results were obtained using only 25 processors for the fluid domain for a computational time of 15.75 h for the same physical time. At this point, the LBM-FEM framework still outperforms the SPH-FEM one even with

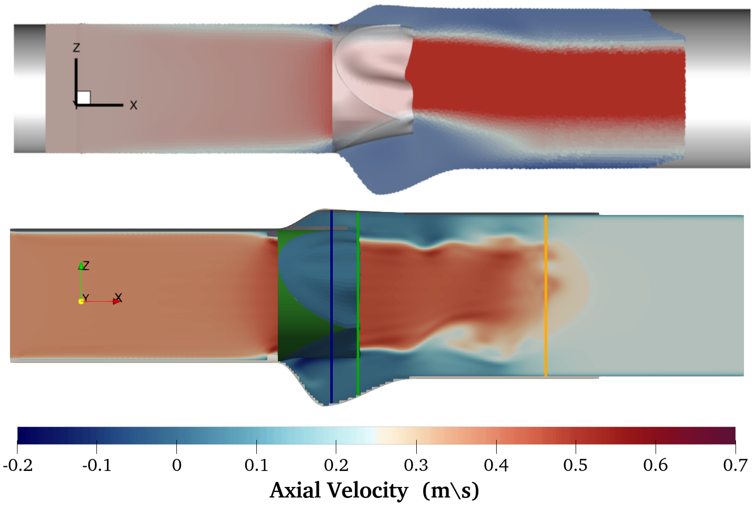

Fig. 2. 3D pulsatile flow through bio-prosthetic aortic valve: Comparison of axial velocity fields between SPH-FEM (top plot) and LBM-FEM (bottom plot) at $t = 0.26$ s

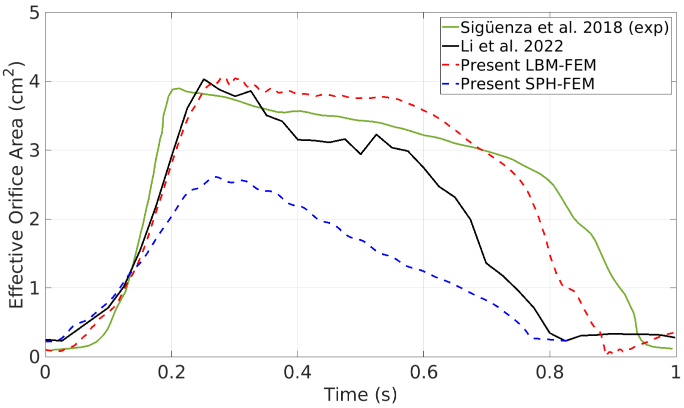

Fig. 3. 3D pulsatile flow through bio-prosthetic aortic valve: EOA comparison

larger SPH time-steps. A complete and accurate study on CPU costs will be carried out for both tools and will be presented at the workshop.

IV. CONCLUSION

The present LBM-FEM formulation provides accurate results, both in terms of velocity fields and EOA, and appears as an efficient tool to simulate the blood flow through an aortic valve, despite difficulties to calculate the stress applied on the valve. While very promising results have been obtained for the simulation of the fluid flow, the SPH-FEM coupling suffers from an underestimation of the forces acted upon the solid structure. A more accurate computation of theses forces is under achievement and will be presented during the workshop in order to reach the same level of accuracy observed for LBM results. Doing so, the resulting formulation could benefit from both the advantages of SPH for the fluid simulation and a robust and accurate FSI. An in-depth investigation of CPU costs will also be conducted, as computational times are still restraining for such SPH simulations, and solutions to overcome this issue are yet to be explored, i.e the so-called 'Adaptive Particle Refinement (APR)' method.

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