Framework for uncertainty quantification of wave-structure interaction in a flume

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I. INTRODUCTION

Water-borne hazards like tsunamis, storm surges, and flooding can cause major damage. The 2004 Indian Ocean tsunami, for example, created waves up to 20 meters high and killed over 250,000 people. Similar disasters include the damage to ports and infrastructure during the 2010 Chile and 2011 Tohoku tsunamis, as well as the destruction caused by Hurricane Katrina's storm surge in 2005. These events, once considered rare, are happening more often due to climate change and increased activity in coastal areas. This makes it crucial to understand their impact on buildings and infrastructure to make them stronger. There is also growing interest in clean energy, including offshore wind turbines and hybrid energy platforms.

Wave flume experiments are highly valuable for coastal engineers to study how waves interact with structures. These experiments have been used to investigate many coastal engineering challenges, such as beach erosion, the design of breakwaters and seawalls, the impact of waves on offshore structures, and sediment transport [2], [4], [5]. However, building and running wave flumes can be costly and time-consuming, especially when testing multiple scenarios. Careful planning and scaling are also needed to ensure the results are relevant to real-world conditions. To save costs, particularly during the design stage, numerical models can help design flume experiments and verify the accuracy of the models themselves.

In an effort to replicate the experiments by computer simulations, digital wave flumes were developed using numerical methods. Common methods and tools that have been adopted include potential flow solvers (e.g., OceanWave3D [1]), shallow water solvers (e.g., GeoClaw [6], AdCirc [7]), finite volume (FVM) solvers (e.g., OpenFOAM [8]–[10]), and mesh-less solvers (e.g., smoothed particle hydrodynamics (SPH) [11]–[14]). A comprehensive review of developments in WSI, encompassing both numerical methods and experiments, are provided in the recent work of Huang [15].

This study combines three numerical methods—Smoothed Particle Hydrodynamics (SPH) for wave modelling, the Finite Element Method (FEM) for structural response, and Latin Hypercube Sampling (LHS) for structural parameter sampling. While these methods are all well-established methods with extensive development in their respective fields, the novelty of this work lies in coupling these components to address uncertainties in the system structural response. Moreover, most of the works where SPH has been coupled with a structural solver have been limited to the adoption of monolithic rigid structures (cubes, cylinders etc.) or flexible beam structures. In this study, SPH is coupled offline with OpenSees [16]. The reason for employing offline coupling is that, while it is possible to use SPH for the entire WSI modelling process, the SPH structural model is computationally expensive and time-consuming compared to using the OpenSees model. Additionally, within the context of the SPH framework, UQ has seen limited integration previously. More specifically, this work explores UQ in the structural response under wave loading. While the framework has been presented to study the WSI in flume, the framework itself is generalisable. For example, when addressing variations in structural parameters, such as shape, size, or orientation. These variations are integral to the processes of structural design and optimisation. By coupling SPH with OpenSees and integrating UQ analysis, the computational efficiency of the modelling process can be substantially enhanced, particularly when evaluating multiple structural configurations. Such uncertainties can have a critical impact in real-world decisions. The developed framework can be directly employed to explore these applications. This work further serves as a precursor to potential developments of a surrogate model-driven digital wave flume that could enable the incorporation of more extensive probabilistic wave conditions in the future.

II. METHODOLOGY

This study uses DualSPHysics [17] with GPU acceleration to model wave generation and propagation in the numerical flume. Specifically, the Wendland C^2 kernel function, as implemented in DualSPHysics [19], is employed. Following the discretisation technique outlined above, the weakly compressible SPH formulation in DualSPHysics [17] is applied to derive the governing equations. The value of α is set to 0.01 to ensure numerical stability in wave and wave-loading applications [12]. The initial speed of sound, $c_0 = c(\rho_0)$, is defined within the incompressible limit of $Ma = 0.1$ by setting $c_0 = 10\sqrt{gH}$, where H represents the initial water height at rest. The symplectic position Verlet scheme [18] is used for time integration. A dynamic boundary condition (DBC) [20] is applied throughout the work.

Uncertainty quantification (UQ) is combined with structural dynamic analyses to estimate the probabilistic response of structures. This work considers the widely adopted Latin Hypercube Sampling (LHS) as it facilitates an efficient exploration of the parameter space by systematically partitioning the parameter ranges into equi-probable intervals and selecting samples such that one sample is placed in each interval of each parameter dimension [21]. The resulting LHS samples ensure a more uniform and representative distribution of samples across all dimensions, effectively preventing sample clustering.

Without loss of generality, the n input parameters can be given to be $p_i \ \forall \ i = 1, 2, .., n$ and characterised by a specified range and divided into q equi-probable intervals or bins. The interval width can be given to be

$$\text{Interval width} = \frac{\text{Max}(p_i) - \text{Min}(p_i)}{q} \tag{1}$$

where, $\text{Max}(p_i)$ and $\text{Min}(p_i)$ are maximum and minimum value possible for the parameter p_i respectively, q denotes the desired number of samples. The $q-$random permutations of the integers from 1 to q (inclusive), for each parameter p_i, are generated using a random permutation function, namely the Fisher-Yates shuffle algorithm [22].

The LHS matrix L is further created, which contains the LHS samples in domain of $[0, 1]^n$. The dimension of L is $q \times n$ and each row corresponds to a sample point in the n-dimensional input space, with $L_i = (L_{i1}, L_{i2}, .., L_{in})$, where L_{ij} is the value of the j-th parameter in the i-th sample. The LHS sample matrix is constructed using the generated random permutations and the intervals. The elements of the LHS matrix L are mapped to the actual parameter values based on the intervals created using

$$p_{ij} = \text{Min}(p_j) + (L_{ij} - 1) \times \text{Interval width} \tag{2}$$

where, p_{ij} is the mapped value of the j-th parameter for the i-th sample.

Given the randomness in the individual structural properties (here depicted as a vector of properties) $\mathbf{p}_{\text{str}}$ and external forces $\mathbf{F}$, the probability density function (PDF) of the structural responses of interest like Root Mean Squared Acceleration (RMSA), $\mathbf{A}$, is computed as

$$\xi(\mathbf{A}) = \int \xi(\mathbf{A}|\mathbf{p}_{\text{str}}, \mathbf{F}) \ \xi(\mathbf{p}_{\text{str}})\xi(\mathbf{F}) \ \mathrm{d}\mathbf{p}_{\text{str}} \ \mathrm{d}\mathbf{F} \tag{3}$$

where $\xi(\cdot)$ denotes the PDF and $\xi\left(\mathbf{Y}|\mathbf{X}\right)$ denote the conditional PDF of any $\mathbf{Y}$ given $\mathbf{X}$. Note here that $\mathbf{p}$ is used to denote the set of all properties where uncertainties can exist. Some examples include Youngs' modulus, Poisson ratio etc. Without additional randomness considered, the first term in the integral becomes a Dirac-delta function, $\xi(\mathbf{A}|\mathbf{p}_{\text{str}}, \mathbf{F}) = \delta(h(\mathbf{p}_{\text{str}}, \mathbf{F}) = \mathbf{A})$, where $h(\mathbf{p}_{\text{str}}, \mathbf{F})$ represents the system equation, i.e. a combination of wave flume and structural dynamic simulation model. The PDF is approximated by LHS and kernel density estimation (KDE) as

$$\hat{\xi}(\mathbf{A}) = \frac{1}{q}\sum_{i=1}^{m} K_h(\mathbf{A} - A_i) = \frac{1}{q}\sum_{i=1}^{m} K_h(\mathbf{A} - h(p_{\text{str,i}}, F_i)) \tag{4}$$

where $[p_{\text{str,i}}, F_i]$ represents i-th sample obtained from LHS and A_i is corresponding structural analysis outcome. $K_h(\cdot)$ is a kernel function and this work uses, the commonly used, Gaussian basis function [23].

III. Discussion and Results

Probabilistic structural analysis is performed to identify the range of structural responses considering the uncertainties in structural properties or the wave. The former includes yield strength, young's modulus, weight of different part of structure and more intricate characteristics. The latter can be attributed as a variation in the wave forces, i.e wave heights, itself. While the probabilistic analysis is computationally expensive compared to deterministic simulations, it provides more reliable estimates for practical engineering applications [24].

Thus, An OpenSees beam-column model is used to represent a two-storey structure consisting of column, beam, and girder sections, constructed using steel W-sections. The loading on the structure is derived from the wave loading history applied at each centroid node (i.e. storey level). This force history is obtained from the WSI results simulated by DualSPhysics, which are validated by experimental and OpenFOAM results from the literature. A forward UQ method, namely LHS [23], and different structure variables are used for sample generation and aid in quantifying the uncertainties. This work employs the developed NHERI SimCenter UQ engine [23]. The relationships between root mean square accelerations (RMSA) and the random variables are illustrated in Figure 1.

the variations in RMSA versus random variables show a clear distinct behaviour for each of the different wave heights considered. This further establishes that the initial wave heights significantly influence RMSA. The column, beam and girder stiffness, yield strength all show a weak nonlinear relationship with the RMSA. As expected, the overall RMSA decreases as Youngs' modulus value increases.

IV. Conclusion

The work provides a comprehensive numerical framework to estimate probabilistic structural response under wave loading conditions in a wave flume. However, the developed method is general and applicable to other geometries as well. Even for a single wave flume geometry, structural variables significantly influence the structural response under different wave conditions. Deterministic analyses have a limited ability to consider multiple variables simultaneously, which makes it difficult to analyse combined effects that can easily lead to structural failure. Therefore, uncertainty quantification remains essential to extend the interpretation of wave flume results to real-world conditions, even though conducting uncertainty quantification analysis increases computational cost.

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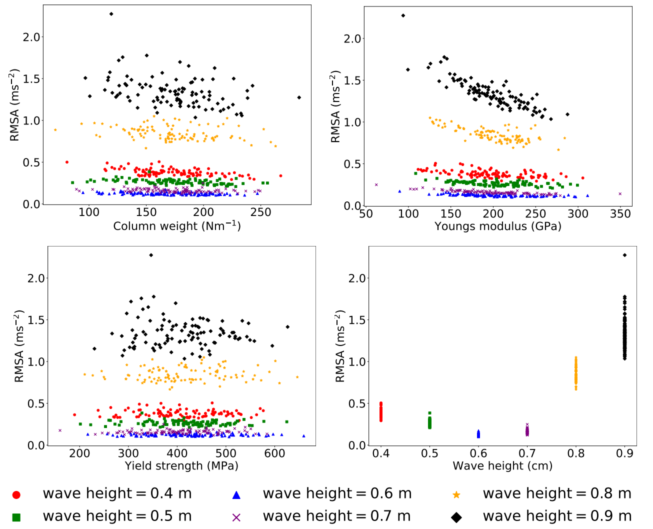

Fig. 1. Two-storey structure's root mean square accelerations (RMSA) in x-direction obtained using OpenSees plotted vs. (a) W-section weight per length for columns (left upper), (b) Young's modulus (right upper), (c) Yield strength (left lower), (d) Wave height (right lower)

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