

Numerical simulation of long-term sloshing flows using an enhanced SPH model

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I. INTRODUCTION

Liquid sloshing is a common issue in ocean, aerospace, and nuclear engineering. The motion of sloshing liquid in a tank is a highly nonlinear phenomenon, characterized by significant fluctuations of the free surface, breaking, and gas-liquid mixing. This process is also accompanied by substantial impact forces, which may cause structural failure of tank walls. From the viewpoint of numerical simulation, simulating violent sloshing is computationally challenging because of the highly nonlinear fluid motion, unknown free surface and the coupling between fluid and structure.

The WCSPH method has been obtaining significant developments and extensive application in modelling sloshing flow due to its Lagrangian and meshfree nature. However, WCSPH suffers from several numerical issues. One is the spurious pressure fluctuations by the combined actions of several factors such as the fully-explicit algorithm and the collocated spatial discretization. Although different pressure stabilization techniques have been proposed (e.g. diffusive terms and Riemann solvers), the lower-frequency acoustic waves with a length scale larger than the radius of the kernel need to be effectively mitigated [1]. Khayyer et al. [1] developed a velocity divergence error mitigating scheme (VEM) to enhance the divergence-free velocity field. However, the diffusion term only handles the instantaneous velocity divergence error, but the residual errors in the velocity divergence fields accumulate over time. A more effective velocity divergence error cleaning method, especially for long-term simulations, is the incorporation of the hyperbolic/parabolic divergence cleaning (HPDC) scheme [2], which advects and diffuses velocity divergence error by designated hyperbolic and parabolic terms. Recently, Zhan et al. [3] introduced the combination of VEM and HPDC to address the issue of accumulated velocity divergence errors. Despite its effectiveness in minimizing the velocity divergence errors, the velocity divergence field may not become completely noise-free due to the associated approximations of the velocity divergence field, especially in the vicinity of truncated kernel domains.

The second issue corresponds to the volume non-conservation during simulation. Within the WCSPH framework, due to the absence of constraints on fluid incompressibility, the constant kernel summation density in the fluid is not maintained, resulting in volume non-conservation. This issue becomes more

pronounced in long-term simulations. Moreover, the widely adopted particle shifting technique has been proven to exacerbate non-physical expansions and contractions of fluid volume. Recently, Khayyer et al. [1] proposed the Volume Conservation Shifting (VCS) scheme, which compensates for numerical errors related to the deviation of particle density with respect to the reference density. However, the effectiveness of the VCS in maintaining volume conservation over long-term sloshing simulations has yet to be examined.

To address the issues above, this study employs the recently published open-source code DualSPHysics+ [3] to simulate long-term liquid sloshing in a tank. Based on the concept of projection-based particle methods, a novel VEM scheme (abbreviated as VEM⁺) is proposed to mitigate the velocity divergence errors in the vicinity of truncated kernel domains more effectively. The primary focus is on evaluating the performance of the combined VEM⁺ and HPDC schemes in mitigating errors in velocity divergence in violent sloshing cases, as well as assessing the effectiveness of the VCS scheme in ensuring volume conservation during long-term sloshing simulations.

II. NUMERICAL METHODS

A combination of the δ -SPH [4,5] and the Riemann-based SPH [6] is adopted, leading to the δ R-SPH. Meanwhile, the combination of VEM and HPDC is proposed to achieve a better velocity divergence error cleaning in both instantaneous and accumulated errors. The discretized governing equations as follows:

$$\frac{D\rho_a}{Dt} = -\rho_a \sum_b \mathbf{u}_{ab} \cdot \nabla_a W_{ab} V_b + D_a \quad (1)$$

$$\frac{D\mathbf{u}_a}{Dt} = -2 \sum_b m_b \left(\frac{p^*}{\rho_a \rho_b} \right) \nabla_a W_{ab} + \mathbf{g} + \mathbf{a}_a^{\text{VEM}^+} - \nabla \psi_a \quad (2)$$

where D is the density diffusion term [7], p^* is the pressure at the intermediate state, ψ is a new scalar field and is related to the velocity divergence error [2]. $\mathbf{a}_a^{\text{VEM}^+}$ is the VEM⁺ acceleration component, which is obtained by the combination of accelerations corresponding to the pressure term $p_a^{\text{VEM}^+}$ and $p_a^{\text{VEM}^{\text{II}}}$. In the work by Khayyer et al. [1], $p_a^{\text{VEM}^{\text{II}}}$ is computed based on the deviation of the velocity divergence field from zero at time step $k-1$ [1], as:

$$p_a^{\text{VEM}^1} = c_s^2 d \rho_a = c_s^2 \Delta t \left(\frac{D\rho}{Dt} \right)_a^{k-1} = -\rho_a c_s^2 \Delta t \langle \nabla \cdot \mathbf{u} \rangle_a^{k-1} \quad (3)$$

Inspired by the concept of projection-based particle methods, a secondary corrective pressure $p_a^{\text{VEM}^{\text{II}}}$ for further minimization of velocity divergence error can be derived by projecting the instantaneous velocity field onto a velocity divergence space, and by explicitly solving a corresponding PPE expressed as:

$$\left\{ \nabla \cdot \left(\frac{\nabla p^{\text{VEM}^{\text{II}}}}{\rho} \right) \right\}_a^{k+1} = \frac{\langle \nabla \cdot \mathbf{u} \rangle_a^k}{\Delta t} \quad (4)$$

The OPS (Optimized Particle Shifting) scheme [8] is applied to properly implement particle shifting at and in the vicinity of free surfaces by effectively eliminating the shifting normal to the free surfaces. To ensure density invariance, the so-called VCS scheme [1] is also implemented in the DualSPHysics+. A pressure term corresponding to the VCS scheme, i.e., p^{VCS} , is computed by explicitly solving a PPE (Poisson Pressure Equation) with the volume conservation error as the source term, as:

$$\left\{ \nabla \cdot \left(\frac{\nabla p^{\text{VCS}}}{\rho} \right) \right\}_a^{k+1} = \frac{\tilde{\rho}_0 - \tilde{\rho}_a^k}{\tilde{\rho}_0 \Delta t^2} + \frac{1}{\Delta t^2 \tilde{\rho}_0 c_s^2} p_a^{\text{VCS},k+1} \quad (5)$$

Note that p^{VCS} leads to additional particle acceleration and, hence, to the volume conservation shifting vector $\delta \mathbf{r}_a^{\text{VCS}}$, as follows:

$$\delta \mathbf{r}_a^{\text{VCS}} = \Delta t^2 \left(-\frac{1}{\rho_a} \langle \nabla p^{\text{VCS}} \rangle_a^L \right) \quad (6)$$

III. NUMERICAL INVESTIGATIONS

Validations in terms of accuracy and efficiency are shown via classical benchmarks. Firstly, the sloshing under rotational excitation [9] is simulated to verify the accuracy of the proposed schemes in violent free-surface fluid flows. The comparison of the free-surface deformations between the experimental photos and the numerical results simulated by $\delta\text{R-SPH-VEM}^+\text{-HPDC-VCS-OPS}$ are shown in Figure 1. The free-surface deformations are in good agreement with the experimental snapshots. Meanwhile, the simulated pressure fields are also smooth during the simulation. Figure 2 shows the time history of the slamming pressure simulated by $\delta\text{R-SPH-VEM}^+\text{-HPDC-VCS-OPS}$. The key features of all numerical results are in the consistent trend with the experimental data although the pressure peaks are slightly over-predicted, which is presumably attributed to the single-phase simulation ignoring the air effect.

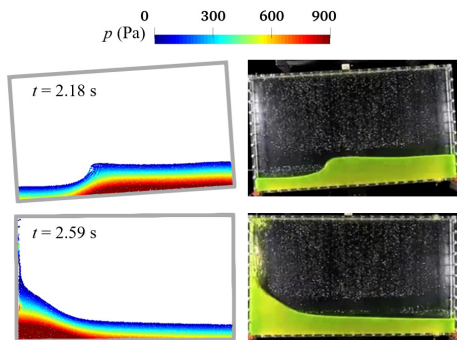


Figure 1. Fluid pressure contours simulated by $\delta\text{R-SPH-VEM-HPDC-VCS-OPS}$ with the corresponding experimental photos [9] at two time instants.

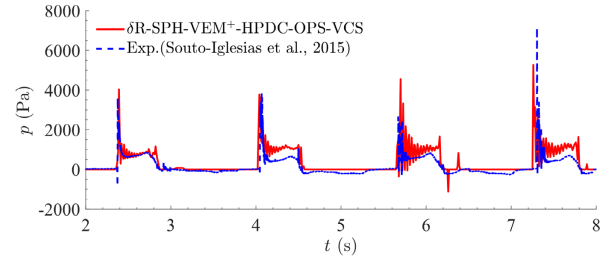


Figure 2. Time history of the simulated slamming pressure along with the experimental data [9].

Secondly, the isothermal sloshing in a circular tank (ISOPE CCP-WSI Blind Test 5) is simulated to validate the efficiency of the proposed models in long-term simulation. The velocity divergence contours at typical time instants are shown in Figure 3. In the results by $\delta\text{R-SPH}$, non-zero velocity divergences of relatively large amplitudes occur in the entire computational domain and their spatial positions change, which correspond to the compression/rarefaction of fluid and the acoustic pressure waves. The adoption of VEM-HPDC effectively reduces the velocity divergence errors across most parts of the flow field and enhances the spatial continuity of the velocity divergence field. However, it can be observed that at $t = 6.7$ s and 9.5 s, velocity divergence noises, near the free surface at the center and the side walls of the tank are not eliminated. From column (c), it is evident that such noises are further mitigated by the incorporation of VEM^+ , demonstrating that VEM^+ is more effective in reducing velocity divergence noises in the vicinity of truncated kernel domains.

To illustrate the effectiveness of the introduced schemes in maintaining volume conservation. The kernel summation density field produced by the schemes without and with VCS are studied. As shown in Figure 4 (a), the density fields by $\delta\text{R-SPH}$ exhibit obvious density increase inside the fluid domain with time (with magnitude being 2.3%). Evident noises and discontinuities can be observed in the free surface vicinity region. From Figure 4 (b), it

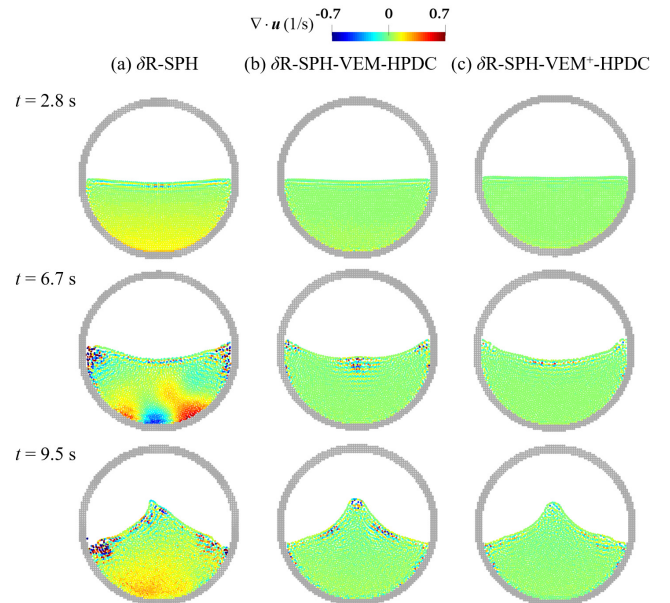


Figure 3. Velocity divergence contours at $t = 2.8$ s, 6.7 s and 9.5 s simulated by (a) $\delta\text{R-SPH}$; (b) $\delta\text{R-SPH-VEM-HPDC}$; (c) $\delta\text{R-SPH-VEM}^+\text{-HPDC}$.

can be seen that the density non-conservation errors and noises are effectively mitigated by the implementation of VCS. Figure 4 (c) shows the results simulated by δR -SPH-OPS. At $t = 7.4$ s, the improvements in particles distribution at the free surface region is obvious by adopting OPS. However, the density field exhibits evident decrease in the whole domain after long-term simulation, which correspond to the unphysical fluid volume expansion and are induced partially by the particle shifting technology. The comparison of the column (c) and (d) contours shows that the VCS scheme effectively mitigates the density errors and achieves better volume conservation. The combination of OPS and VCS can improve particle distribution while avoiding volume non-conservation issues.

The time histories of the free surface elevation measured at the center of the tank are presented in Figure 5 (a). As can be seen, the curves obtained by δR -SPH-OPS-VCS model show periodic evolutions, whereas the curves obtained by δR -SPH-OPS gradually float as the simulation time increases. This confirms that, with the VCS scheme, the fluid expansion introduced by particle shifting is prevented, and therefore the problems of the non-physical free-surface rising-up can be addressed. Figure 5 (b) shows the time histories of the normalized potential energy, the potential energy predicted by δR -SPH-OPS consistently increases over time, showing an 80% increase from the initial value at $t = 50$ s. Following the implementation of the VCS scheme, volume conservation is maintained, which in turn ensures the conservation of potential energy.

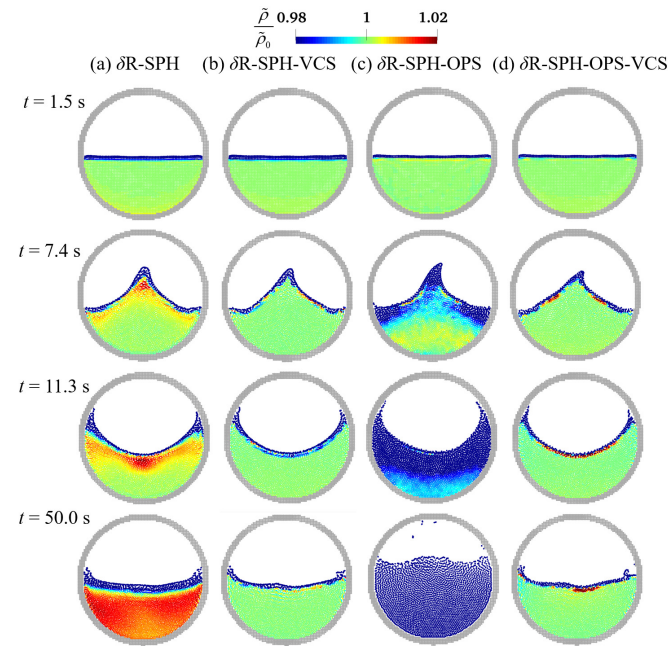


Figure 4. Particle density contours evaluated by kernel summation at $t = 1.5$ s, 7.4 s, 11.3 s and 50.0 s predicted by: (a) δR -SPH; (b) δR -SPH-VCS; (c) δR -SPH-OPS; (d) δR -SPH-OPS-VCS

IV. CONCLUSIONS

In this study, an enhanced velocity divergence error mitigating scheme (VEM^+) is developed and integrated with the recently published SPH open-source code DualSPHysics+ for free surface flows. The accuracy of the proposed model is validated via a violent liquid sloshing benchmark case. Based on

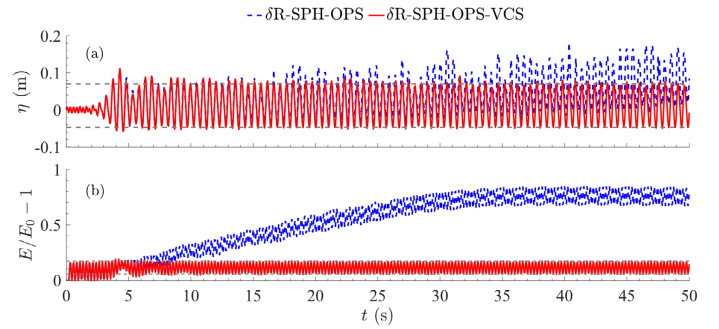


Figure 5. (a) time histories of the free surface elevation at the center of the tank. (b) time histories of the normalized fluid potential energy.

this model, a long-term simulation of liquid sloshing in a circular tank is conducted. Results demonstrate that the combination of VEM^+ and HPDC significantly reduces velocity divergence errors and improves the spatial continuity of the velocity divergence field. Compared with VEM , VEM^+ more effectively mitigates velocity divergence noises in the vicinity of truncated kernel domains. The adoption of OPS enhances particle distribution at the free surface but induces unphysical volume expansion of particles, leading to a rise in water level and potential energy during long-term simulations. The application of VCS ensures global density invariance and improves fluid volume conservation. By enhancing density invariance and velocity divergence-free conditions, better numerical resolution of the continuity equation in incompressible liquid sloshing flow simulations is achieved.

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