A HIGH ACCURATE MESHLESS METHOD FOR FREE SURFACE FLOWS

José Maciñeira Ortiz de Urbina*
Group of Numerical Methods in Engineering (GMNI)
University of A Coruña (UDC)
A Coruña, Spain
jose.macineira@udc.es

Luis Ramírez
Group of Numerical Methods in Engineering (GMNI)
University of A Coruña (UDC)
A Coruña, Spain

ABSTRACT

Nowadays, advances in engineering and CFD have led us to face problems with more complex geometries and higher accuracy needs.

Traditionally, mesh-based methods have been employed to solve this CFD problems, but with today's computational facilities, meshless methods have become very popular for dealing with large deformation or complex geometries problems, like free-surface or wave breaking problems.

In this work we present a high–accurate meshless method based on the use of Moving Least Squares (MLS) approximants [2] for the resolution of the Weakly-Compressive Navier Stokes equations [3] on an ALE approach. The method was developed to overcome some of the grand challenges [4] of traditional Lagrangian SPH methods, like boundary conditions, accuracy and the treatment of the free surface.

The accuracy and robustness of the proposed method are demonstrated by performing tests cases.

I. DRAWBACKS AND GRAND CHALLENGES

As established by the SPH community in [4], SPH has some fields where research is needed in order to compete with other more established methods, such as FDM, FEM or FVM. These main fields are grouped in five Grand Challenges.

- GC1: Convergence, consistency and stability.
- GC2: Boundary Conditions.
- GC3: Adaptivity.
- GC4: Coupling to other methods.
- GC5: Applicability to industry.

Although the Grand Challenges do not cover some important fields of research as the simulation of turbulence, they sum up the main goals that the method should face up to establish in the scientific community.

In this work, it will be cover mainly three of them, which are GC1, GC2 and GC3.

As it will be seen, the SPH-ALE will be used to face up the accuracy and the stability of the method [9], and the MLS approximants will be employed to face up the consistency [6] and the boundary conditions.

On the other hand, since the SPH-ALE will be use as a base of the presented formulation with MLS approximants, the scheme will have many possibilities to coupling with other methods as FVM and FDM.

II. GOVERNING EQUATIONS

In fluid dynamics, when we talk about the governing equations we refer, in the most part of the cases, to the Navier-Stokes equations. This set of Non-linear equations is composed of the continuity equation, the momentum equation and the energy equation and they describe the behaviour of a viscous flow under some external forces, based on the conservation of mass, momentum and energy in the domain of study

In the following, the set of equations will be introduced through an integral approach of the conservation laws in an *ALE description*, with the main approaches over the pressure field.

A. Weakly compressible approach

The physical behaviour of fluid is studied through the fundamental state variables in fluid dynamics, which are density, velocity, energy and pressure. The first three of them are obtained through the fundamental conservation laws of physics, namely, the conservation of mass, conservation of linear momentum and the conservation of energy. The last one is obtained relating it to the density field.

If an incompressible fluid is being studied and compressible effects could be neglected, the density field is set to be constant. This approach is based on the resolution of the Poisson equation for the pressure field, imposing null divergence of velocities in the continuity equation, and is usually used in Coastal and Hydraulic Engineering. On the other hand, in problems where compressible effects should not be neglected, even though the fluid could be considered as incompressible, the complete Navier-Stokes equations must be solved. This involves the need of a Thermodynamic Equation of State for the pressure field, which relates it with the rest of state variables. It is worth mentioning that when using an incompressible approach, the simplest state equation possible is being used, which is ρ = constant.

The choice of the state equation in the compressive approach depends on the characteristics of the fluid and the flow, so it would be necessary to use one equation of state per fluid involved in the flow. This kind of problems are usually related with astrophysics and aeronautics, but they are also common in Engineering problems as the study of breaking waves, wave impact or the analysis of fluid-structure interaction.

Although water usually is modelled as incompressible, to take advantage of the two previous approaches, a Weakly-Compressible hypothesis will be considered [8]. This means that density variations are under 1%. If we assume that there are not heat sources and the internal energy is constant (barotropic fluid), the Thermodynamic Equation of State depends only on the density, and the pressure field could be easily obtained. This approach avoids the resolution of the Poisson equation for the pressure field, while considerer cuasi-incompressible fluids.

B. *ALE framework*

In this work, a generalization of the well known Lagrangian and Eulerian description is employed, this is the ALE description (Arbitrary Lagrangian Eulerian), where the domain changing rate $\boldsymbol{\omega}$ is introduced.

The ALE framework is based on the use of the Reynolds Transport Theorem, which gives us a link between the Lagrangian or Material description and the Spatial or Eulerian one.

$$R.T.T. \rightarrow \frac{d}{dt}\int_\Omega \Phi \, d\Omega = \int_\Omega \frac{\partial \Phi}{\partial t}\, d\Omega + \oint_\Gamma \Phi \boldsymbol{\omega}^T \boldsymbol{n}\, d\Gamma$$

Where the surface integral describes the variation of the domain through the domain changing rate $\boldsymbol{\omega}$ previously introduced.

With and ALE configuration, it is possible to combine the best features of the material and spatial description, while recovering them by imposing zero domain changing rate (Eulerian description) or establishing it equal to the fluid velocity (Lagrangian description).

C. *Set of equations*

The final set of the Navier-Stokes equations is introduced in its ALE approach, by using the Transport Operator.

$$\boldsymbol{L_w}(\cdot) = \frac{\partial(\cdot)}{\partial t} + \boldsymbol{\nabla} \cdot [\boldsymbol{\omega} \otimes (\cdot)]$$

Obtaining the final set.

$$\boldsymbol{L_w}(\boldsymbol{U}) = -\boldsymbol{\nabla} \cdot (\boldsymbol{F_\omega}) + \boldsymbol{\nabla} \cdot (\boldsymbol{D}) + \boldsymbol{S}$$

Where $\boldsymbol{F_\omega}$ are the convective fluxes, $\boldsymbol{D}$ are the diffusive ones, $\boldsymbol{S}$ is the vector of source terms and $\boldsymbol{U}$ is the vector of conservative variables.

III. Proposed Method

The derivation of the method is developed in [1], introducing MLS approximants as shape functions and substituting the classical kernel.

$$\frac{d(V_i \boldsymbol{U}_i)}{dt} + \sum_{j=1}^{n_i} \left[\frac{1}{2}(\boldsymbol{F}_j + \boldsymbol{F}_i) - \boldsymbol{F}_i\right]\left(\oint_\Gamma N_i N_j \cdot \boldsymbol{n}\, d\Gamma - V_j \nabla \mathrm{N}_{ij} + V_i \nabla N_{ji}\right) = V_i \boldsymbol{S}_i$$

Where N represents the MLS approximants, $\boldsymbol{F}$ the numerical fluxes and V the volume of the particle.

IV. Test Cases

In this point, the proposed method is implemented to perform some of the most common test cases of the bibliography.

A. *Taylor Green Vortex*

The first test case is the classical Taylor Green Vortex, where the decaying of the kinetic energy is analysed, comparing it with the exact solution.

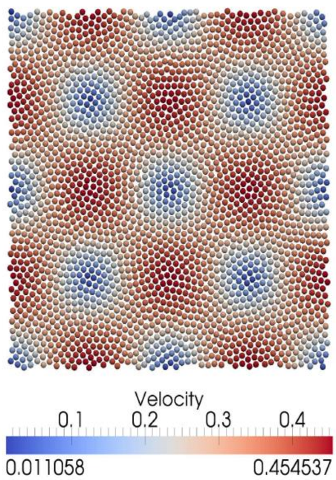

Figure 1. Taylor-Green vortex. Re = 100 with 2500 particles. [5]

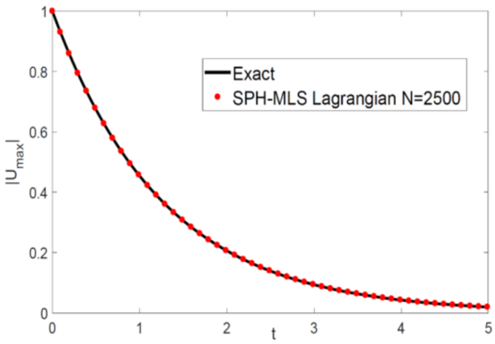

Figure 2. Exact – Numerical solution [5]

B. Laminar flow past a 2D cylinder

The second test case is the studying of a non-steady viscous flow past a two-dimensional cylinder, analysing the drag, lift and Strouhal coefficients obtained.

The radius of the cylinder is R = 0.5 and the free-stream Mach number is 0.1.

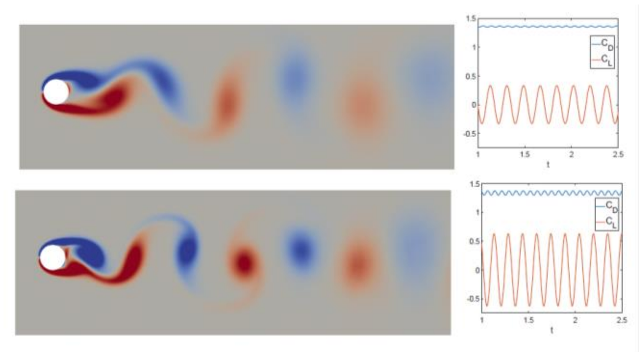

Figure 3. Laminar flow past a 2D cylinder: Re = 100 and Re = 185. [1]

The results of the obtained drag, lift and Strouhal number are compared with results obtained from the literature.

TABLE I.- ANALYSIS OF RESULTS.

	Re = 100			Re = 185		
Method	$\overline{C_D}$	C_l	S_t	$\overline{C_D}$	$C_{l,rms}$	S_t
Present Method	1.3587	±0.3325	0.1653	1.3398	0.4447	0.1937
Liu et al. FVM	1.35	±0.339	0.165	-	-	-
Rajani et al. FVM	1.3353	-	0.1569	-	-	-
Marrone et al. SPH	1.36	±0.24	0.168	-	-	-
Ng et al. IB	1.368	±0.36	-	-	-	-
Constant et al. IB	1.37	-	0.165	1.379	0.427	0.198
Vanella and Balaras IB	-	-	-	1.377	0.461	-
Guilmineau FDM	-	-	-	1.287	0.443	0.195
Lu and Dalton FDM	-	-	-	1.31	0.422	0.195
Liu et al. SPH	-	-	-	1.372	0.427	-
Sun et al. FEM	-	-	-	1.363	-	0.196
Liu and Hu IB	-	-	-	1.289	0.451	0.197

V. FINAL CONTRIBUTION

Finally, the adaptation of the proposed method to free surface flows will be introduced, comparing its results with others obtained from the literature [6], [7].

BIBLIOGRAPHY

[1] Eiris, A. Et al. From mesh to meshless: *A generalized meshless formulation based on Riemann solvers for Computational Fluid Dynamics.*

[2] L. Ramírez, A Eirís, I Couceiro, J París, X Nogueira. *An Arbitrary Lagrangian-Eulerian SPH-MLS method for the computation of compressible viscous flows.* JCP, 111172, 2022

[3] A Eirís, L Ramírez, J Fernández-Fidalgo, I Couceiro, X Nogueira. *SPH-ALE Scheme for Weakly Compressible Viscous Flow with a Posteriori Stabilization.* Water, 13 (3), 245, 2021.

[4] R. Vacondio, C. Altomare, M.D. Leffe, X. Hu, D.L. Touzé, S. Lind, J.C. Marongiu, S. Marrone, B. Rogers, A. Souto-Iglesias, *Grand challenges for smoothed particle hydrodynamics numerical schemes,* Comput. Part. Mech. 8 (2020) 575–588.

[5] Ramirez, L; Eiris, A.; Couceiro, I.; París, J.; Nogueira, X., *An arbitrary Lagrangian-Eulerian SPH-MLS method for the computation of compressible viscous flows.*

[6] L. Cueto-Felgueroso, G. Mosqueira, I. Colominas, F. Navarrina, M. Casteleiro, *Análisis de formulaciones numéricas SPH para la resolución de problemas de flujo con superficie libre,* SEMNI-2002.

[7] Antona, R.; Vacondio, R.; Aversani, D.; Righetti, M.; Renzi, M., *A WENO SPH scheme with improved transport velocity and consistent divergence operator.*

[8] Monaghan, J. J. (1994). *Simulating free surface flows with SPH.* Journal of Computa tional Physics, 110(2), 399–406.

[9] J.P. Vila, *On particle weighted methods and smooth particle hydrodynamics,* Math. Models Methods Appl. Sci. 09(02) (1999) 161–209.