Extension of the RHOD-SPH framework: towards a fully Adaptive Particle Refinement technique

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Multi-resolution technique represents a hot topic within the SPH research community. In the present work, we propose to extend the RHOD-SPH scheme derived in [1] to multi-resolution technique. Fine particles are concentrated in zones of interest and, thanks to the RHOD-SPH framework, no buffer regions are required. A dynamic refinement procedure is used to maintain fine particles where needed, e.g. in the free-surface region or close to solid boundaries. The formulation is validated for complex free-surface flows, showing a good agreement with both reference and single resolution results.

I. Introduction

For many applications the number of SPH particles is huge, resulting in prohibitive computational times and memory requirements. To tackle this problem, a multi-resolution technique is required. In mesh-based numerical methods Adaptive Mesh Refinement has been developed for decades (see *e.g.* [2, 3]). By contrast, this topic is more recent in SPH and still represents an active research topic within the SPH research community.

In [4] a multi-resolution technique was studied for water entry of solid wedges. The finest particles were concentrated close to the impact zone with a geometric progression of the particles size (3% of growth rate) towards the tank border. Later in [5], splitting and coalescing procedures were developed to dynamically adapt the particles size during the simulations, allowing to drastically reduce the number of particles required for internal viscous flows around obstacles.

Further, a multi-domain decomposition technique was developed (see *e.g.* [6, 8, 9]), consisting in using different predefined geometrical domains (circular or rectangular) characterized by different particle resolutions. The particles resolution within each geometrical domain was maintained using splitting and coalescing procedures in the transition zones. Two strategies have been employed to treat these transition zones, depending on the SPH operators used and consequently on the targeted applications. Indeed, for free-surface flows, conservative SPH operators were required to intrinsically fulfill the dynamic free-surface boundary condition [10]. Therefore, buffer regions were employed in [8, 9] to avoid inaccurate SPH approximations in the transition zones, implying (i) specific treatments, (ii) algorithm complexity and (iii) restriction on the geometrical domain shape. By contrast, for internal flows, accurate (but not conservative) SPH operators can be used and, as shown in [6], no buffer regions are required with these operators.

Recently [1], an SPH formulation has been derived whose main features are (i) hybrid pressure gradient approximation *i.e.* accurate operator inside the fluid domain and conservative operator in the free-surface region, (ii) regular particles distribution maintained thanks to Particle Shifting Technique (PST), (iii) use of quasi-Lagrangian formulation to take into account the PST in the governing equations and (iv) numerical diffusion obtained by means of Riemann solver. This formulation (called hereinafter RHOD-SPH) has been validated for both internal and free-surface flows, but exclusively for single-resolution cases. In the present work, we propose to extend the RHOD-SPH framework to multi-resolution. Precisely, the Riemann diffusion terms are generalized to deal with multi-resolution and the PST is adapted; details will be given during the presentation. Then, this formulation is validated in a multi-resolution context, using a dynamic refinement procedure to concentrate the finest particles in zones of interest. By contrast to multi-domain decomposition technique [6, 8, 9], the zones of interest are not limited to predefined circular or rectangular geometrical domains, since no buffer regions are required thanks to the SPH operators used. This framework is studied considering thin refinement zones close to the free-surface and/or solid boundaries on dedicated benchmarks. Note that the coalescing procedure is let to future work, as other refinement criteria based on fluid flow features.

II. The RHOD-SPH scheme for multi-resolution

In this section the RHOD-SPH scheme [1] is recalled and its extension to multi-resolution is explained. It reads:

$$\begin{cases} \dfrac{d\rho_i}{dt} & = -\rho_i \nabla \cdot \boldsymbol{u}_i - \rho_i \nabla \cdot \boldsymbol{\delta u}_i + \nabla \cdot (\rho_i \boldsymbol{\delta u}_i) + \Theta^{\rho}_{i,Rie} \\ \rho_i \dfrac{d\boldsymbol{u}_i}{dt} & = -\nabla P_i + \rho_i \boldsymbol{f}_i + \boldsymbol{F}^{\mu}_i + \boldsymbol{F}^{AD}_i + \nabla \cdot (\rho_i \boldsymbol{u}_i \otimes \boldsymbol{\delta u}_i) + \dfrac{\boldsymbol{\Theta}^{\boldsymbol{u}}_{i,Rie}}{V_i} \\ \dfrac{d\boldsymbol{x}_i}{dt} & = \boldsymbol{u}_i + \boldsymbol{\delta u}_i, \quad V_i(t) = m_i / \rho_i(t), \quad P_i = c_0^2(\rho_i - \rho_0), \end{cases} \tag{1}$$

where d/dt is the quasi-Lagrangian derivative, ρ the fluid density, P the pressure, V the volume, m the mass, $\boldsymbol{u}$ the fluid velocity, $\boldsymbol{\delta u}$ the shifting velocity, $\boldsymbol{f}$ the external volume forces, $\boldsymbol{F}^{\mu}$ the viscous forces, $\boldsymbol{F}^{AD}_i$ the Acoustic damper forces [7], $\Theta^{\rho}_{i,Rie}$ and $\boldsymbol{\Theta}^{\boldsymbol{u}}_{i,Rie}$ the diffusive terms obtained by means of Riemann solver, $\boldsymbol{x}$ the position of material point, and the subscript i refers to the particle i. Pressure and density fields are linked through $P_i = c_0^2(\rho_i - \rho_0)$ with c_0 chosen as $c_0 \geq 10 \max\left(U_{max}, \sqrt{(\Delta p)_{max}/\rho}\right)$. The pressure gradient approximation reads:

$$\boldsymbol{\nabla} P_i = \begin{cases} \mathbb{L}_i \sum_j (P_j - P_i)\boldsymbol{\nabla}_i W_{ij} V_j & \text{if } i \in \mathcal{I} \ \& \ \Gamma_i \geq 0.95 \\ \sum_j (P_j + P_i)\boldsymbol{\nabla}_i W_{ij} V_j & \text{otherwise} \end{cases} \quad (2)$$

with $\mathbb{L}_i$ the renormalization matrix. For SPH discretization of the other spatial differential operators in Eq. (1), the interested reader is referred to [1, 7].

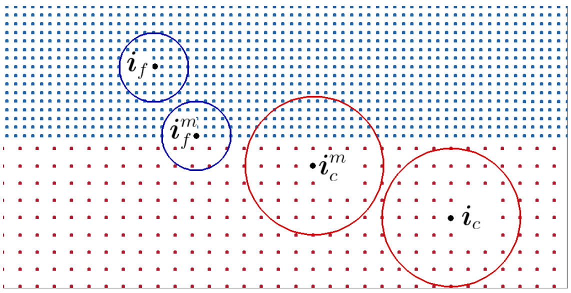

Fig. 1. Sketch of kernel supports for multi-resolution. $\boldsymbol{i}_c$ and $\boldsymbol{i}_f$ stand for coarse and fine particles respectively interacting with particles of same Δx, while $\boldsymbol{i}_c^m$ and $\boldsymbol{i}_f^m$ interact with particles of different Δx.

Regarding the multi-resolution, each particle i has its own kernel support radius R_i depending on a constant ratio set to $R_i/\Delta x_i = 4$ in this work. The set of neighborhood particles of i is then defined as $\mathcal{D}(i) = \{j : \|\boldsymbol{x}_j - \boldsymbol{x}_i\| < R_i\}$, as sketched in Fig. 1.

III. Refinement procedure & criteria

In the present framework, the refinement procedure for a particle i of size Δx_i consists in a splitting in 2^D particles of size $\Delta x_i/2$ with D the spatial dimension. In a first attempt only 2D cases are treated, coarse particles are then split into 4 fine particles. The refinement pattern is chosen to minimize the inter-particles distance once the refinement procedure performed. A minimum size Δx_{min} is fixed prior to the simulation, and a particle i that requires refinement is chosen using the following specific criteria:

1) i interacts with a "grand-daughter" particle, *i.e.* if $\exists j \in \mathcal{D}(i) : \Delta x_j = \Delta x_i/4$.
2) i is an isolated particle: if the neighborhood particles of i are mainly of size $\Delta x_i/2$, i requires refinement. Precisely the criterion reads: $\sum_{j:\Delta x_j = \Delta x_i} W(\boldsymbol{x}_j - \boldsymbol{x}_i)V_j < 2\,W(0)V_i$.
3) the minimum size cannot be crossed, *i.e.* $\Delta x_i/2 > \Delta x_{min}$.
4) i belongs to the free-surface region, *i.e.* i interacts with a free-surface particle.
5) i is close a solid boundary, *i.e.* the kernel support of i intersects a solid panel.

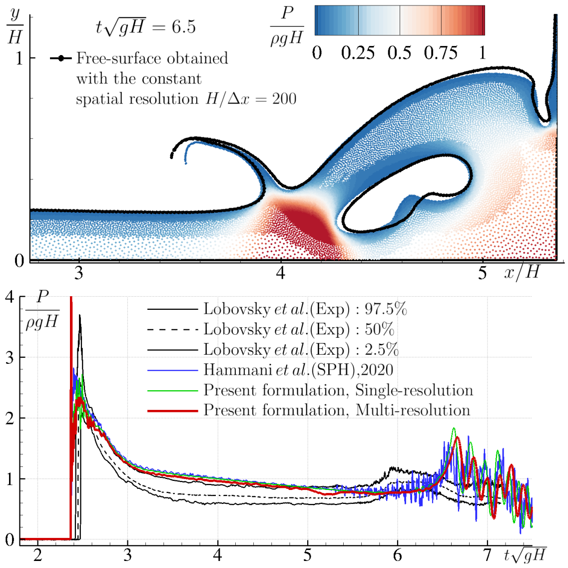

Fig. 3. Dam-break test case. Top: pressure field obtained at $t\sqrt{gH} = 6.5$ and comparison of the free-surface shape to single-resolution. Bottom: temporal evolution of the pressure at sensor 1 location. Comparison of the present multi-resolution solution with reference results and single-resolution.

Criteria (1) $\rightarrow$ (3) are activated in any case. By contrast, criteria (4) and (5) can be activated depending on the targeted application and, if several bodies are involved, the criterion (5) can be activated only on selected bodies. Thanks to the criterion (1), it creates a thin zone of size $2\,R_i$ in which particles are all of same size Δx_i, as sketched in Fig. 2.

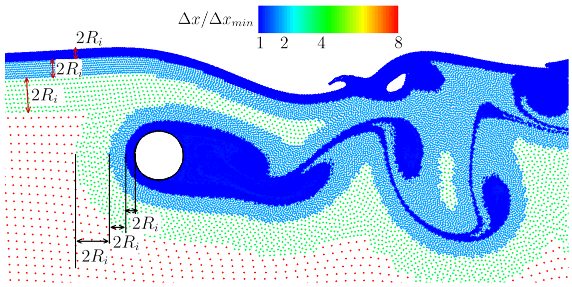

Fig. 2. Example of spatial resolutions obtained for a flow past an immersed cylinder when criteria (1) $\rightarrow$ (4) are activated. Criterion (5) is activated around the cylinder and disabled at the bottom part of the tank.

IV. Numerical Discussion

The present framework is validated for different benchmarks. The refinement is activated (i) exclusively in the free-surface region (dam-break test case in Fig. 3), (ii) exclusively close to solid boundaries (wedge impact test case in Fig. 4) and (iii) both in the free-surface region and close to solid boundaries (flow past an immersed cylinder test case in Fig. 5). The fluid flow solutions obtained are smooth, even in transition zones. Furthermore, the results obtained are in good agreement with both single-resolution and reference results.

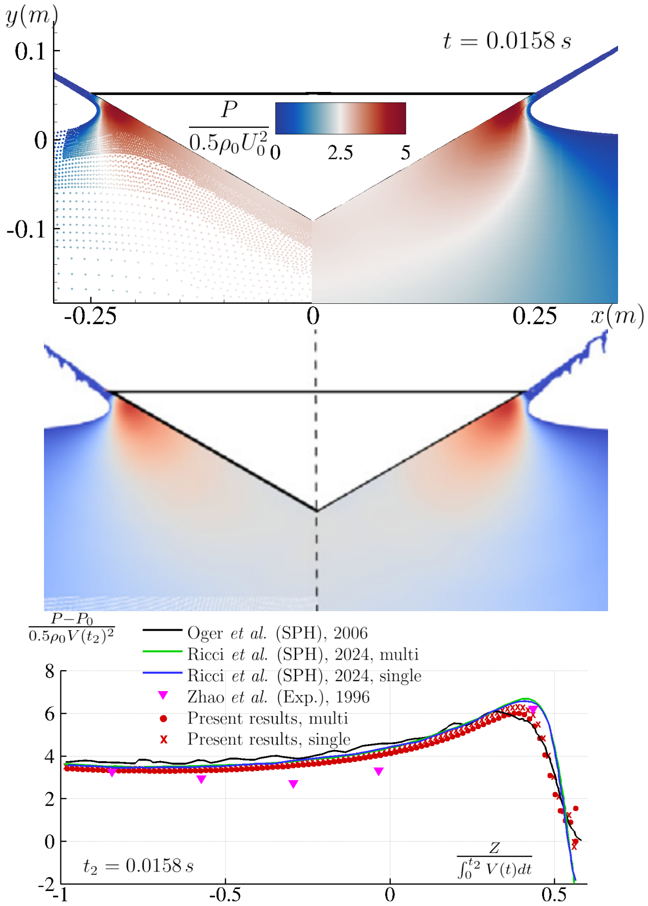

Fig. 4. Impact of a wedge on free-surface. Top plot: Pressure field obtained with the present formulation; left multi-resolution; right single-resolution. Center: Pressure field obtained by Ricci *et al.* [9]; left multi-resolution; right single-resolution. Bottom: pressure measured along the wedge surface compared to reference results.

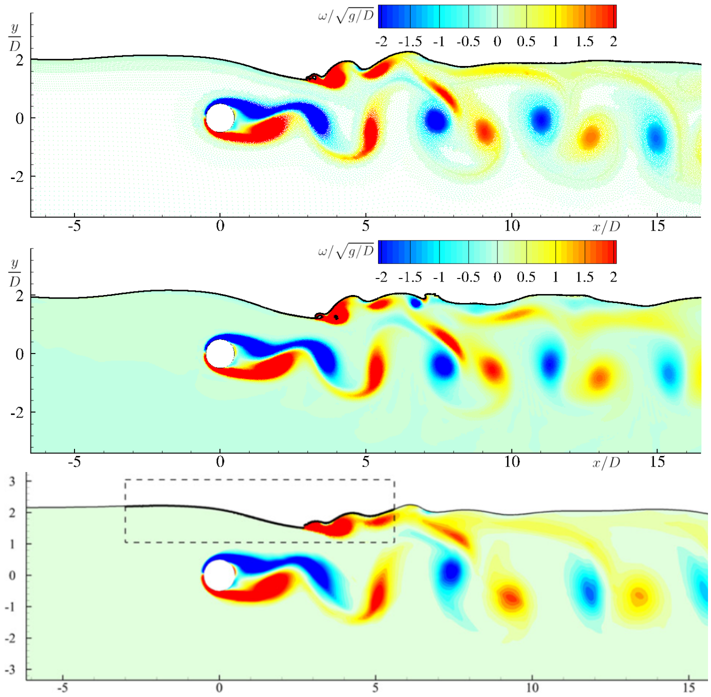

Fig. 5. Flow past an immersed cylinder. Vorticity field obtained with the present framework with multi-resolution (top) and single-resolution (center) compared to the one obtained using a coupling between SPH and Finite Volume method (bottom part). For the coupled results, the SPH method is used inside the dashed rectangle while Finite Volume is used elsewhere.

V. Conclusion

In the present work, the RHOD-SPH framework has been adapted to deal with multi-resolution, with a dynamic refinement procedure. The formulation has been validated for complex free-surface flows, and with interface reconnexion.

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