Capturing violent and turbulent sloshing dynamics using SPH modelling

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I. Introduction

In this study, five different smoothed particle hydrodynamics (SPH) models are used to test the capability of addressing the challenges of violent sloshing flows characterized by high free-surface fragmentation and turbulent behavior. A Large Eddy Simulation (LES) framework is adopted for all five models. The comparative analysis demonstrates the improved performances for the last model which adopts the combination of renormalized pressure gradient, particle shifting technique, and an adaptive numerical diffusive term added only on the continuity equation. This combination is labeled as $\delta^\star$ -SPH and the main advantages include: (i) the resolution of small-scale turbulence, (ii) a more accurate assessment of sloshing energy dissipation, (iii) a reduction in volume conservation errors, and (iv) the absence of tensile instability, establishing it as a robust scheme for studying highly turbulent sloshing flows.

II. Brief recall of the SPH models adopted

The Navier-Stokes equations for weakly-compressible liquid are discretized within a Quasi-Lagrangian SPH model as below:

$$\begin{cases} \dfrac{d\rho_i}{dt} &= -\rho_i \langle \mathrm{div}(\boldsymbol{u}_i + \boldsymbol{\delta u}_i)\rangle + \langle \mathrm{div}(\rho_i \boldsymbol{\delta u}_i)\rangle + \mathcal{D}_i^\rho \\ \rho_i \dfrac{d\boldsymbol{u}_i}{dt} &= \langle \nabla p\rangle_i + \boldsymbol{F}_i^V + \boldsymbol{f}_i + \langle \mathrm{div}(\rho_i \boldsymbol{u}_i \otimes \boldsymbol{\delta u}_i)\rangle + \mathcal{D}_i^{\rho u} \\ \dfrac{d\boldsymbol{r}_i}{dt} &= \boldsymbol{u}_i + \boldsymbol{\delta u}_i,\; V_i(t) = m_i/\rho_i(t),\; p_i = c_0^2(\rho_i - \rho_0), \end{cases} \tag{1}$$

where $\langle \nabla p\rangle_i$, $\boldsymbol{F}_i^V$ and $\boldsymbol{f}_i$ are, respectively, the pressure, the viscous and the body forces acting on the particle i and $\mathcal{D}_i^\rho$ and $\mathcal{D}_i^{\rho u}$ are the numerical diffusivity terms adopted to stabilize the scheme (see *e.g.* [1]). The formulation of the terms $\langle \nabla p\rangle_i$, $\mathcal{D}_i^\rho$ and $\mathcal{D}_i^{\rho u}$ changes with the selected numerical scheme, as described in the following text. The viscous forces are evaluated with a sub-grid model and are described in detail in [2]. The vector $\delta \boldsymbol{u}$ is the shifting velocity related to the Particle Shifting Technique (PST) adopted to regularize the spatial distribution of the particles during their motion. Since the particles are moving with modified velocity $(\boldsymbol{u} + \delta \boldsymbol{u})$ and the above equations are written in an Arbitrary-Lagrangian-Eulerian framework. For this reason, the continuity, and the momentum equations contain terms with spatial derivatives of $\delta \boldsymbol{u}$. The specific law chosen for $\delta \boldsymbol{u}$ is the same reported in [3]. The smoothed divergence operators of (1) are given in [4]. The spatial gradients are approximated through convolution summations with a kernel function W_{ij}. A C2-Wendland kernel is adopted in the present work.

A. Pressure forces model

Regarding the smoothed pressure gradient, two different approaches are considered:

$$\begin{cases} \langle \nabla p\rangle_i^C &= \displaystyle\sum_j \left(p_i + p_j\right) \nabla_i W_{ij} V_j\,, \\ \langle \nabla p\rangle_i^L &= \mathbb{L}_i \displaystyle\sum_j (p_j - p_i)\, \nabla_i W_{ij} V_j \end{cases} \tag{2}$$

where $\mathbb{L}_i$ is the renormalization tensor (see *e.g.* [1]).

The first smoothed operator $\langle \nabla p\rangle_i^C$ is classically used in the conservative SPH models (see *e.g.* [5]). The smoothed operator $\langle \nabla p\rangle_i^L$ can reproduce linear function, therefore is more accurate than $\langle \nabla p\rangle_i^C$ and it avoids the so-called tensile instability [6]. However, it is quite sensible to the particles' spatial distribution. To avoid ill-conditions for the tensor $\mathbb{L}$, $\langle \nabla p\rangle_i^L$ can be used only far enough to the free-surface region, and particle shifting is mandatory for practical applications. For this reason, a third smoothed operator is defined as:

$$\langle \nabla p\rangle_i^\star = \begin{cases} \langle \nabla p\rangle_i^L & \text{if } i \in \mathcal{I} \,\&\, \Gamma_i \geq 0.95 \\ \langle \nabla p\rangle_i^C & \text{otherwise} \end{cases} \tag{3}$$

where the subset $\mathcal{I}$ consists only of the inner fluid particles, *i.e.* those particles whose kernel support does not intersect the free surface. For this purpose, it is necessary to identify the particles belonging to the free-surface [7]. Finally, the second condition $\Gamma_i := \sum_i W_{ij} V_j > 0.95$ ensures that particles belonging to free surfaces that approach each other during liquid impact events are not seen as interior particles.

B. Diffusive terms

It is well known in the literature that the SPH schemes need to be stabilized by introducing numerical dissipation to avoid an extremely noisy pressure field. A traditional way to achieve stabilization is using a Riemann solver to evolve the solution in time. In particular, in this work, the model described in [8] was followed, where the mass of the particles does not vary in time and the acoustic approximation is adopted with a slope limiter considered through a MUSCL (Monotone Upstream Scheme for Conservation Laws) procedure to increase the order of the scheme (see [3]). A simplified version of the Riemann-SPH is the so-called δ-SPH proposed by [9] and extended in the LES framework in [10] where only a diffusive term is added to the continuity equation.

C. The considered five SPH models

Five SPH schemes are considered in the present work. The differences between those models are recalled in Table I based on the use or not of the PST and the presence of not of numerical diffusive terms, and finally which formula is embedded for the pressure forces. The $\delta^\star$-SPH is the novel model proposed in the present work. In the next section, it will be shown that it is the one that presents the best results also in terms of the solution of the vorticity field.

	PST	$\mathcal{D}_i^\rho$	$\mathcal{D}_i^{\rho u}$	$\langle \nabla p \rangle_i$
Standard SPH	NO	0	0	$\langle \nabla p \rangle_i^C$
δ-SPH	NO	$\Theta_{i,\delta}^\rho$	0	$\langle \nabla p \rangle_i^C$
Riemann-SPH	NO	$\Theta_{i,Rie}^\rho$	$\Theta_{i,Rie}^{\rho u}$	$\langle \nabla p \rangle_i^C$
RHOD-SPH	YES	$\Theta_{i,Rie}^\rho$	$\Theta_{i,Rie}^{\rho u}$	$\langle \nabla p \rangle_i^\star$
$\delta^\star$-SPH	YES	$\Theta_{i,\delta}^\rho$	0	$\langle \nabla p \rangle_i^\star$

TABLE I
THE FIVE SPH MODELS CONSIDERED.

D. Evaluation of the slosh dissipation

The energy balance in the noninertial frame of reference (NiFoR) moving with the tank is :

$$[\mathcal{E}_K + \mathcal{E}_P](t) - [\mathcal{E}_K + \mathcal{E}_P](t_0) - \mathcal{W}_{NF} = Q_V + Q_N \tag{4}$$

On the left-hand side of Eq. (4), $\mathcal{E}_K$ and $\mathcal{E}_P$ are the kinetic and potential energy and $\mathcal{W}_{NF}$ is the work related to the non-inertial forces. Conversely, the right-hand side of the energy balance (4) includes both the dissipation due to the real and turbulent viscosity Q_V, while Q_N is the numerical dissipation taking into account the effect of the numerical diffusion terms and the particle displacement $\delta \boldsymbol{u}$. The numerical dissipation is not negligible during fluid impact events, where mechanical energy is converted to compressible energy, which is dissipated by the numerical diffusion terms. Following [11], it can be decomposed as:

$$Q_V = Q_\omega + \mathcal{E}_{FS}, \quad Q_\omega := -\int_{t_0}^{t} \int_\Omega (\mu + \mu_T)\, \|\nabla \times \boldsymbol{u}\|^2 \, dV\, dt\,. \tag{5}$$

where Q_ω is the dissipation power due to enstrophy while $\mathcal{E}_{FS}$ is a power related to the viscous deformation of the fluid domain Ω due to the movement of the free surface.

III. Test-case description

The present study examines violent heave sloshing flow in a partially filled rectangular tank through tests and numerical simulations using single-phase SPH models. This test case was also investigated experimentally (described in detail in [12]). The tank measures L, D and W cm and is filled to 50% of its volume, $H = D/2$, with water. Experimentally, the tank is mounted on six springs, oscillating at a frequency of $f_0 = 6.51$ Hz with an exponentially decreasing amplitude over 25 cycles due to friction and slosh dissipation. The maximum vertical displacement is $2A = 1.14L$. The sloshing flow persists longer due to the high Reynolds number, see [13]. Intense oscillations cause significant deformation and fragmentation of the free surface and turbulent flow (see Figure 1). The reference energy is set as $\Delta E = \rho L H W (A\Omega)^2/2$, where $\Omega = 2\pi f_0$. The experiments were numerically simulated using a single-phase SPH solver, which aligned well with the experimental data ([3], [13]).

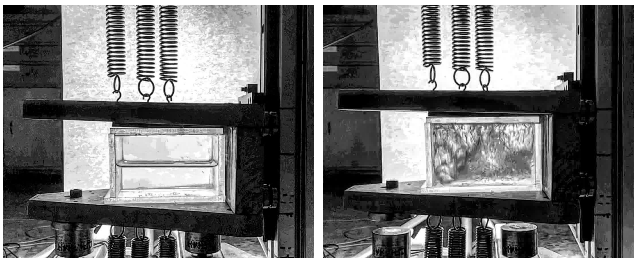

Fig. 1. Snapshots of the experimental campaign, see [12]

IV. Results and discussion

Figure 2 illustrates the vorticity field computed using the δ^*-SPH model, at the maximum spatial resolution $N = H/\Delta r = 800$, at time $t = 3.52T$, corresponding to the most energetic phase of the flow. Multiple scales of vorticity are evident, with distinct eddies generated by the reconnection of the free surface as the tank descends.

The time histories of the enstrophy dissipation Q_ω evaluated by five different models are shown in Figure 3. This plot highlights the capability of the δ^*-SPH model to accurately capture this dissipation component, which remains consistently smaller in magnitude when computed with other SPH formulations.

Finally, Figure 4 presents the time histories of total slosh dissipation as predicted by the δ^*-SPH model in both 2D and 3D frameworks. In the 3D case, the comparison with the experimental data of [12] demonstrates excellent agreement, validating the effectiveness of the proposed model in handling violent free-surface flows. The same figure also shows that the dissipation estimates obtained in the 2D framework are remarkably close to the experimental data, despite the pronounced three-dimensional nature of the turbulent flow under investigation.

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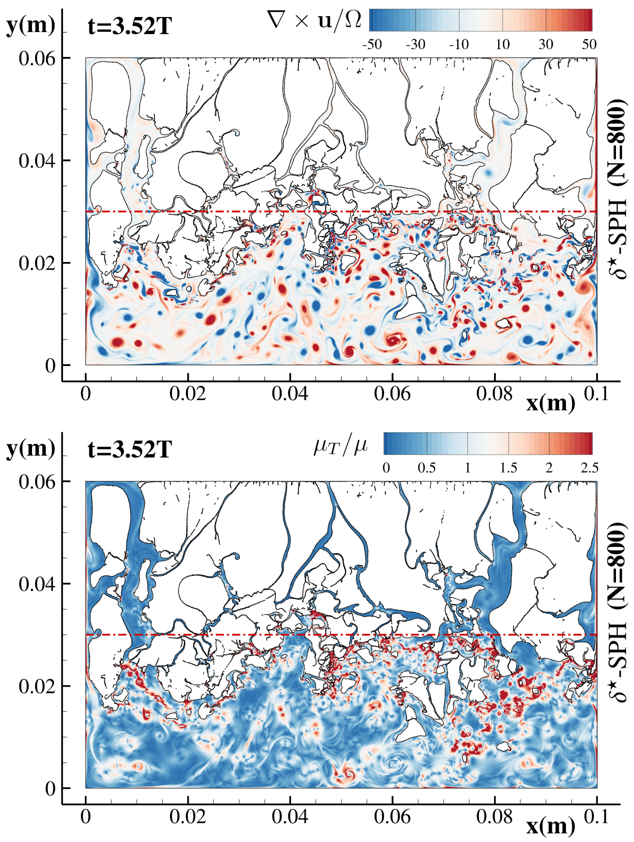

Fig. 2. δ⋆-SPH vorticity (top) and eddy viscosity (bottom) fields at time $t = 3.52T$.

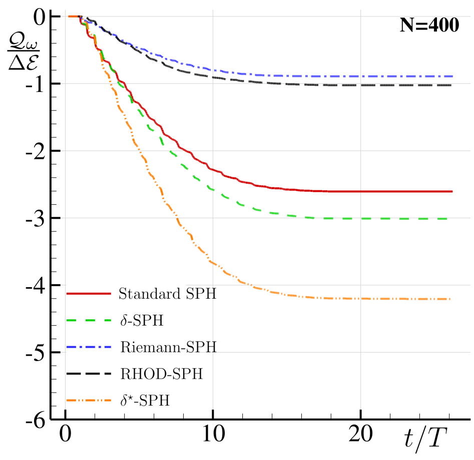

Fig. 3. Enstrophy slosh dissipation, comparison between the different SPH models.

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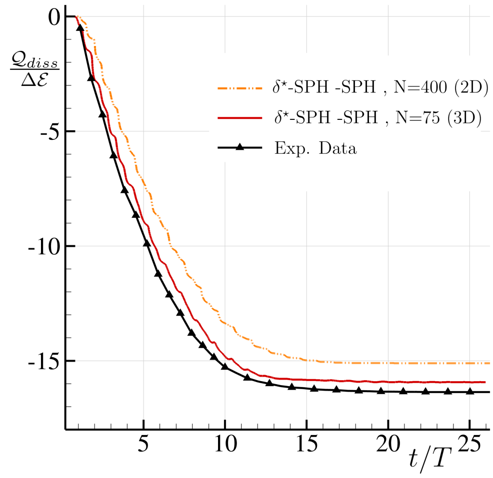

Fig. 4. δ⋆-SPH slosh dissipation in 2D and 3D framework, comparison with the experimental data by [12].

by Siemens Digital Industries Software.

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