A coupled WCSPH-MSDEM model for combined motion of a block contact with solid-wall boundary

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A coupled WCSPH and modified MSDEM model is developed to simulate the motion of arbitrarily shaped discrete blocks under unsteady flow and solid boundary effects. A new normal force algorithm aligns the force direction with the boundary's surface normal, while a friction model based on static friction law and Coulomb theory improves accuracy. The model is validated using theoretical and demonstration cases, including a hollow square's rolling and sliding under plunging waves.

I. INTRODUCTION

The stability of armor blocks on rubble mound breakwaters is a critical concern in coastal engineering. These blocks achieve stability through mass, friction, or interlocking, thereby ensuring the overall safety of the breakwaters. However, during extreme weather events, the violent impact of breaking waves can induce complex combined motions of the blocks, including collisions, rolling, and sliding. The failure of the armor block layer often occurs when significant movement of a few blocks compromises the structure's integrity.

The coupling of Lagrangian-based Smoothed Particle Hydrodynamics (SPH) and Multi-Sphere Discrete Element Method (MSDEM) offers significant advantages in simulating complex interactions between discrete blocks and boundaries, such as geometrically intricate blocks or interactions with granular bodies. MSDEM simplifies solid surfaces by representing them with spheres (or discs in 2D), enabling efficient simulation of block interactions. This method has been widely applied in studies of armor block motion. For instance, Ren et al. [1] simulated the behavior of cross-shaped armor blocks on rubble mound breakwaters under regular waves. Yamamoto et al. [2] investigated block extraction behind caisson breakwaters caused by tsunami overflow, while Mitsui et al. [3] analyzed Tetrapod sliding under solitary wave impact. Additionally, mesh-based CFD coupled with MSDEM has been used to study wave-accropode interactions [4].

This study focuses on a coupled WCSPH and modified MSDEM method. By correcting the normal force direction using surface normal vectors and incorporating static friction theory alongside Coulomb's law for tangential force calculations, the modified MSDEM achieves improved stability and accuracy. A series of validation test cases, along with a demonstration case involving wave impacts on a hollow square placed on a slope, are presented to further confirm the model's effectiveness.

II. NUMERICAL METHODS

A. *Maintaining the Integrity of the Specifications*

The WCSPH method governs fluid motion based on the mass and momentum equations, with the equation of state used to compute pressure. To ensure stability in the pressure field, the δ-SPH term [5] is employed, while laminar viscous and Sub-Particle Scale (SPS) terms are used to model fluid viscosity.

$$\frac{d\rho_i}{dt} = \sum_j m_j \boldsymbol{v}_{ij} \nabla_i W_{ij} + 2\delta h c_0 \sum_j \left(\rho_i^D - \rho_j^D\right) \frac{\boldsymbol{r}_{ij}}{\left|\boldsymbol{r}_{ij}\right|} \nabla_i W_{ij} \frac{m_j}{\rho_j} , \quad (1)$$

$$\begin{aligned}\frac{d\boldsymbol{v}_i}{dt} = &-\sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2}\right) \nabla_i W_{ij} \\ &+ \sum_j m_j \boldsymbol{v}_{ij} \frac{4\mu \boldsymbol{r}_{ij} \nabla_i W_{ij}}{\left(\rho_i + \rho_j\right)\left|\boldsymbol{r}_{ij}\right|^2} + \sum_j m_j \left(\frac{\boldsymbol{\tau}_i}{\rho_i^2} + \frac{\boldsymbol{\tau}_j}{\rho_j^2}\right) \nabla_i W_{ij} + \boldsymbol{g}\end{aligned} , \quad (2)$$

$$p_i = \frac{\rho_0 c_0^2}{\gamma} \left[\left(\frac{\rho_i}{\rho_0}\right)^\gamma - 1\right] . \quad (3)$$

The solid boundaries are handled using a modified Dynamic Boundary Particles (DBPs) approach [6], where the density of DBPs is adjusted based on the average density of neighboring fluid particles.

B. MSDEM method

This study evaluates the contact mechanics of a single block interacting with a solid boundary using the MSDEM method. Both the block and the solid boundary are represented by sets of spheres (or discs in 2D), with sphere diameter Δx matching the resolution of the fluid domain in the WCSPH method. Contact forces, including normal and frictional forces, are calculated by summing contributions from individual block spheres, as shown in Fig. 1.

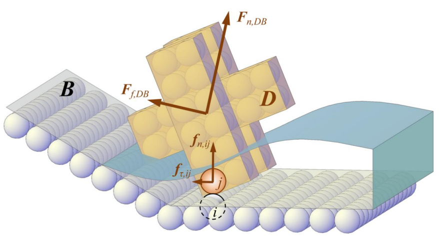

Figure 1. Sketch of discrete block and solid boundary.

The normal force $\boldsymbol{f}_{n,ij}$ of each block sphere i contact with block sphere j is calculated using nonliner Hertzian model resemble to Canelas et al.[7], with its direction following surface normal vector $\boldsymbol{n}_j$ of sphere j according to its relative position to surrounding boundary spheres k, as shown in Fig. 2. The $\boldsymbol{f}_{n,ij}$ is calculated by

$$\boldsymbol{f}_{n,ij} = k_{n,ij}\delta_{n,ij}^{3/2}\boldsymbol{n}_j - c_{n,ij}\delta_{n,ij}^{1/4}\boldsymbol{v}_{n,ij} , \quad (4)$$

where $\delta_{n,ij}$ and $\boldsymbol{v}_{n,ij}$ are normal overlap and relative velocity along $\boldsymbol{n}_j$, and the stiffness and damping constants $k_{n,ij}$ and $c_{n,ij}$ are related to the material properties including Young's modulus, Poisson's ratio and restitution coefficient.

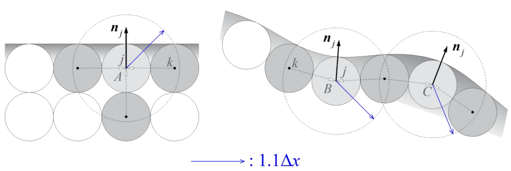

Figure 2. Sketch of surface normal vectors for boundary spheres.

The tangential force, i.e. the friction on each block sphere $\boldsymbol{f}_{\tau,ij}$ is considered based on static friction law and Coulomb theory. As shown in Fig. 3, for friction judgement and calculation, the tangential component $\boldsymbol{F}^*_{\tau,DB}$ of external force exerted on the block D, including mass force, fluid force and normal force from boundary B is firstly needed. The $\boldsymbol{F}^*_{\tau,DB}$ is then distributed to each block sphere i according to the ratio of the magnitude of normal force of sphere i to the total normal force exerted on block D:

$$\boldsymbol{f}^*_{\tau,ij} = \boldsymbol{F}^*_{\tau,DB}\frac{|\boldsymbol{f}_{n,ij}|}{|\boldsymbol{F}_{n,DB}|} . \quad (6)$$

where * means that this variable is obtained before solving the friction. Subsequently, the property of the friction force is determined by comparing the friction judgement is proceeded by comparing the tangential external force $\boldsymbol{f}^*_{\tau,ij}$ with the static friction threshold $\mu|\boldsymbol{f}_{n,ij}|$, and the tangential relative speed $\boldsymbol{v}_{\tau,ij}$ with a minimum speed threshold $c_0/10000$, and the $\boldsymbol{f}_{\tau,ij}$ is solved according to the different properties of the friction.

$$\boldsymbol{f}_{\tau,ij} = \begin{cases} -\boldsymbol{f}^*_{\tau,ij} - c_{\tau,ij}\boldsymbol{v}_{\tau,ij} & , \textit{ if } \left|\boldsymbol{f}^*_{\tau,ij}\right| < \mu\left|\boldsymbol{f}_{n,ij}\right| \textit{ and } \left|\boldsymbol{v}_{\tau,ij}\right| < c_0/10000 \\ -\mu_{DB}\left|\boldsymbol{f}_{n,ij}\right|\dfrac{\boldsymbol{v}_{\tau,ij}}{\left|\boldsymbol{v}_{\tau,ij}\right|} & , \textit{ otherwise} \end{cases} . \quad (7)$$

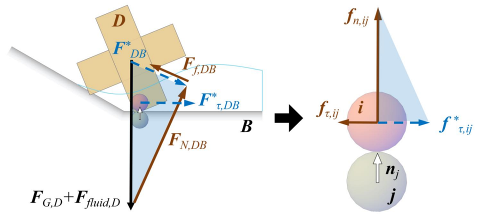

Figure 3. Sketch of tangential external force and tangential contact force of block and block sphere in the modified MSDEM method.

C. Block motion, coupling scheme and numerical scheme

The motion of the discrete block is governed by rigid body dynamics, where Newton's second law is applied to update both its translation and rotation. The coupling of WCSPH and MSDEM method follows the approach proposed by [6], with Newton's third law used to calculate fluid forces on each DBP. An explicit second-order Symplectic scheme is employed as a time-stepping scheme in the above model, with variable time step technique adopted.

III. Validation tests and numerical example

Three test cases are employed to validate the present model. In the first case, as shown in Fig. 4(a). the collision and movement of a ball with radius r=0.1m and mass m=62.83kg on a smooth semi-circular surface with radius R=2.0m are simulated. The initial position of the ball's center is at the specified staring point $(x_0, z_0) = \left(-\sqrt{2}R/2, R/4 - 3\sqrt{2}R/8\right)$m , and after the ball is released, it moves under the influence of gravity and multiple collisions. The calculated trajectory of the ball in t=0-1.8s is shown in Fig. 4(b). It is evident that the original MSDEM results deviated from the theoretical trajectory, while the modified MSDEM results align well with the theoretical solution.

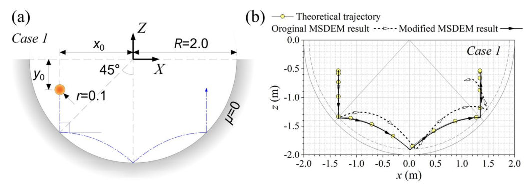

Figure 4. (a)Sketch of a 2D ball impacting a semi-circular boundary at the initial moment (Unit: m), and (b)comparison of ball trajectories from t=0s to t=1.8s for the original and modified MSDEM models.

The second case and the third case include a 2D 0.1×1.0m box with a mass of m=200kg toppling onto a rough plane under the action of an external force, and the box leaning on a vertically wall toppling down under the action of gravity, as shown in Fig. 5(a). The time histories of the velocities of the box's centroids are presented in Fig. 5(b). It is obvious that the original MSDEM results fail to match the theoretical solution due to embedding phenomena during the sliding of the box's vertices along the boundary, while the modified MSDEM results align well with theoretical results. The results demonstrate the effectiveness of the modified MSDEM method in simulating the complex mixed motion of sliding and rotation in a block contacting a solid boundary.

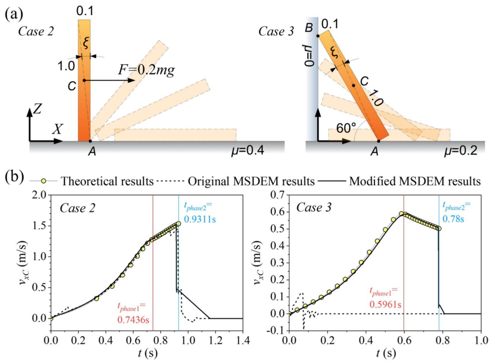

Figure 5. (a)Sketch of a 2D box sliding and rotating on a rough plane and on a corner, with (b)comparisons of centroid velocity for the original and modified MSDEM models, alongside theoretical solutions.

Finally, a demonstration case is provided to further illustrate the stability and robustness of the coupled WCSPH-modified MSDEM model. This case involves a 3D hollow square block with dimensions 0.1m×0.1m×0.06m and mass of m=0.75kg positioned on a rough slope in a numerical flume. The combined rolling and sliding motion of the block under the action of regular waves are simulated. The gradient of the slope is i=1:3, and the friction coefficient between the block and the slope is set as μ=0.55, with water depth d=0.4m and regular wave condition of H=0.14m and T=1.6s.

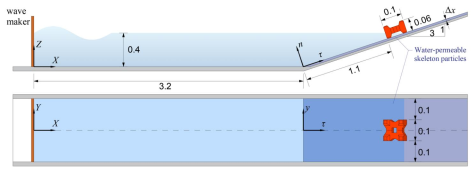

Figure 6. Schematic illustration of the numerical layout of a single hollow square initially positioned on a slope.

The results of the block's motion, including the time histories of sliding and rotation are depicted in Fig. 7, The block exhibits both upward and downward displacement along the slope due to wave action, though its net movement is downslope, with a final displacement of 0.6 m at t=10s. In addition, the block completes two 180° rotation around the Y-axis between t=5.2 and t=7.6s. Fig. 8 presents 4 selected snapshots showcasing the block's rotating behaviors, indicating that the block's rotations are caused by the plunging wave impact or by the rolling flow of the plunging wave.

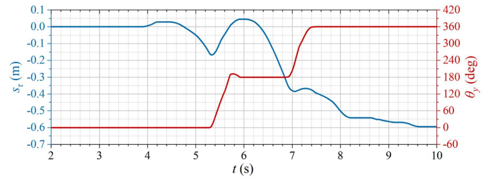

Figure 7. Histories of block's sliding along the slope and rotation angle about the Y-axis.

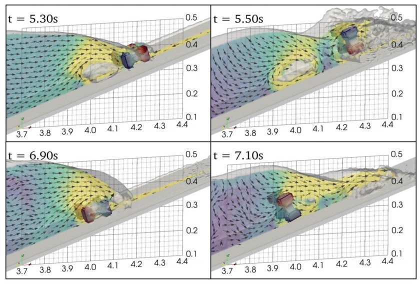

Figure 8. Snapshots of the block's motion at four moments.

IV. Conclusion

A coupled WCSPH-MSDEM numerical model has been developed in this work, based on the open-source DualSPHysics v5.0 code to improve the simulation of the contact of rigid discrete bodies with complex geometries on solid boundaries. The MSDEM method proposed by Canelas et al. [7] is modified to improve its stability to handle contact between discretized blocks and a solid boundary, enabling efficient simulation of collision and sliding behaviors. Currently, the model is undergoing further validation to ensure its stability and accuracy for handling contact between blocks and arbitrarily shaped boundaries, as well as for managing interactions among multiple blocks.

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