Variable resolution SPH with a Local Time Stepping procedure

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I. INTRODUCTION

Variable resolution and adaptivity for Smoothed Particle Hydrodynamics (SPH) are currently listed among the SPHERIC Grand Challenges [1], and they constitute an active field of research with has prompted major efforts in the last decade. Different approaches have been proposed to introduce variable particle size for the SPH method, among which the popular particle splitting-coalescing method [2] and the adaptive particle refinement method [3], [4]. Recently, Ricci et al. [5] presented a novel multi-resolution method based on a domain decomposition, implemented in the open-source code DualSPHysics [6]. This variable resolution approach has been validated against different test cases, spanning from flow past bluff bodies to fluid-structure interaction problems involving complex free-surface interfaces in two and three dimensions [7]. The algorithm has shown it can greatly reduce the number of SPH particles required compared to a uniform-resolution simulation, even when run on GPU architectures.

When executing a variable resolution simulation in SPH, it is common to advance the solution everywhere in the computational domain using the smallest time step that comes from the finest particle resolution, determined from stability criteria. This approach is often inefficient, especially when dealing with a large range of time scales. In astrophysical applications, this issue is addressed by using local time steps, allowing SPH simulations with several billion particles [8], [9]. Only recently, an algorithm based on local time steps [10] has been proposed in the context of engineering applications of fluid dynamics.

In this work, the multi-resolution method developed in [5] is augmented with a Local Time Stepping (LTS) algorithm that defines a hierarchy of temporal levels. Each subdomain resulting from the decomposition of the domain into zones of different resolutions is assigned to a specific temporal level based on the stability condition of the chosen time stepping scheme. The numerical solution is advanced simultaneously through the different temporal levels adopting a synchronization procedure. The LTS algorithm improves the efficiency of the variable-resolution method for multi-scale applications by decreasing the number of time steps needed in zones of lower resolution.

The manuscript is structured as follows: in Section II, a brief summary of the LTS algorithm is presented. Section III shows the validation for flow past a sphere at Reynolds 300. Finally, some conclusions and future work are drawn in Section IV.

II. LOCAL TIME STEPPING ALGORITHM

The Local Time Stepping algorithm is based on the definition of a hierarchy of time levels. Each time level n_i to which subdomain i is assigned is calculated by:

$$n_i = \text{round}\left(\log_2\left(\frac{\Delta t_i}{\Delta t_{\text{min}}}\right) - 1\right) \tag{1}$$

where Δt_i is the time step of subdomain i, determined by the stability condition of the time stepping scheme, and Δt_{min} is the minimum Δt_i across all subdomains. The total number of time levels is set equal to $N = \max(n_i)$.

The global time step $\Delta t_g = \Delta t_N$ is calculated as:

$$\Delta t_g = \Delta t_{\text{min}} \cdot 2^{(N-1)} \tag{2}$$

During a global time step, $n_s = 2^{(n_i-1)}$ smaller time steps are computed for each subdomain i with a local time step given by:

$$\Delta t_{\text{local}} = \Delta t_{\text{min}} \cdot 2^{(n_i-1)} \tag{3}$$

The synchronization between subdomains during the global time step is achieved through a second-order symplectic integrator, and it is achieved via the following operations sequence:

1) Particle interaction in the predictor step (IP).
2) Solution advancement in the predictor step (PRE).
3) Particle interaction in the corrector step (IC).
4) Solution advancement in the corrector step (COR).
5) Buffer particle creation/deletion and interpolation for coupling (VR).

Figure 1 illustrates the procedure for $N = 2$ time levels. The dashed arrows connecting the two time-level lines represent the exchange of information between subdomains, carried out via a corrected SPH interpolation to update the physical values of the particles in the buffer regions. These buffer regions are needed to impose the Dirichlet boundary conditions and close the computational subproblems.

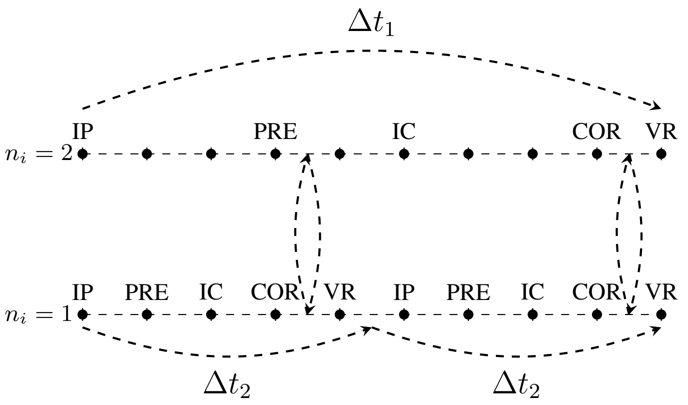

Fig. 1: LTS synchronization procedure with $N = 2$ time levels

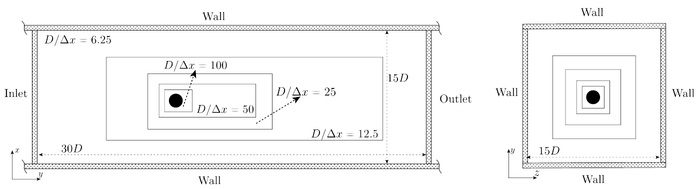

Fig. 2: Longitudinal (left) and cross-sectional (right) views of the computational domain for flow past a sphere

III. Results and Discussion

The flow past a sphere for a Reynolds number of 300 is chosen as a validation case. At this Reynolds, an accurate description of the flow requires a resolution of at least $D/\Delta x \approx 100$ near the sphere to properly capture the boundary layer. For this reason, this flow cannot be simulated in a reasonable amount of time unless a variable resolution approach is chosen. Figure 2 shows a sketch of the computational domain, extending $10D$ upstream and $20D$ downstream of the sphere, in the direction of flow. The domain cross section perpendicular to the flow direction is square, with a $15D$ side length chosen to avoid any boundary disturbance. Four refinement regions are nested one inside the other, with a particle size spanning $D/\Delta x = 6.25$ to $D/\Delta x = 100$, and a total number of SPH particles $N_p \approx 11 \times 10^6$.

The time history of the drag, C_D, and lift, C_L, coefficients obtained with and without the LTS algorithm are shown in Figure 3. The two numerical solutions overlap to a great extent both in average value of the aerodynamic coefficients and oscillation peaks. The same result is noted for the shedding frequency of vortices in the wake, shown in Figure 4. This figure shows a spectral analysis of the lift coefficient, returning a maximum power spectral density at a Strouhal number of St $= 0.133$. Table I summarizes these numerical results, comparing them with reference solutions from other studies available in the literature. A good agreement is observed overall, highlighting the effectiveness of the LTS algorithm for three-dimensional flows.

To further verify the accuracy of the Local Time Stepping algorithm when compared to the original multi-resolution model

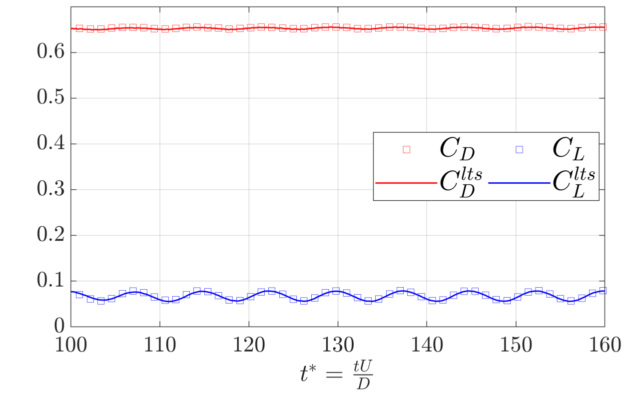

Fig. 3: Time history of drag and lift coefficients for flow past a sphere with variable resolution SPH with and without Local Time Stepping

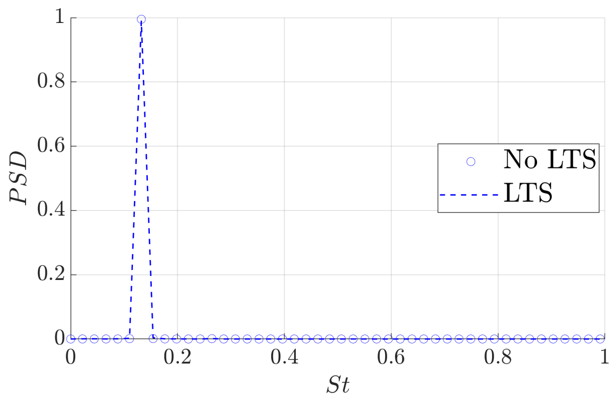

Fig. 4: Normalized power spectral density of the lift coefficient with and without Local Time Stepping

in [5], the average streamwise velocity in the wake centerline is extracted for both approaches and presented in Figure 5. A close agreement is once again noted.

Table II compares the computational performance of the Local Time Stepping algorithm with respect to the original implementation in [5], analyzed after executing both simulations of flow past a sphere at Re $= 300$ on an NVIDIA A-100 GPU card. The employment of LTS yields a reduction in computational time by a factor of 1.74. This improvement can be interpreted by examining the average number of time steps performed by a particle,

TABLE I: Drag and lift coefficients, and Strouhal number for flow past a sphere at Re $= 300$, solved with and without the LTS algorithm, and compared against reference values

	C_D	C_L	St
Present (Local Time Stepping)	0.653	0.068	0.133
Present (without Local Time Stepping)	0.653	0.068	0.133
Johnson and Patel (1999) [11]	0.656	0.069	0.138
Costantinescu and Squires (2000) [12]	0.655	0.065	0.136

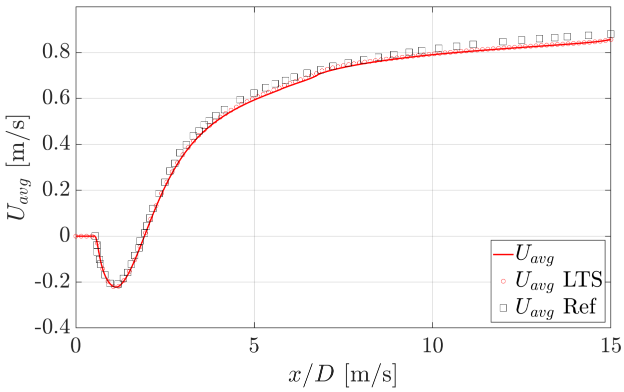

Fig. 5: Comparison of average and root-mean-square values of streamwise velocity along the wake centerline with and without Local Time Stepping against results in [13]

TABLE II: Normalized computational time, T_t, and average number of time steps performed by a particle, $\tilde{N}_{step}$, for flow past a sphere at Re = 300 with and without the LTS procedure

	T_t	$\tilde{N}_{step}$
Present (Local Time Stepping)	1	1
Present (without Local Time Stepping)	1.74	1.85

defined as $\tilde{N}_{step} = \sum_i N_p^i N_{step}^i / \sum_i N_p^i$, where N_p^i and N_{step}^i are respectively the number of particles and the number of time steps for the i-th subdomain. The comparison of $\tilde{N}_{step}$ with and without the LTS procedure shows that $\tilde{N}_{step}/\tilde{N}_{step}^{LTS} \approx 1.85$, which closely matches the observed computational speed-up, demonstrating that the present algorithm is able to approach the theoretical maximum speed-up and introduce negligible computational overhead.

IV. Conclusions

A Local Time Stepping procedure is presented within the framework of variable resolution SPH based on a domain decomposition, originally introduced in [5]. The algorithm defines a hierarchy of time levels that advance the numerical solution simultaneously, following a synchronization procedure described in Section II. The flow past a sphere at Re = 300 is selected as the validation case. Numerical results demonstrate that the proposed LTS algorithm achieves a significant computational speed-up without compromising the accuracy of the numerical solution. Future work will see the application of the LTS algorithm to additional, challenging studies that are found in real-world engineering problems.

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