Towards h/p refinement for Weakly Compressible SPH obtained with Eulerian/Lagrangian coupling

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I. Introduction

One of the research areas to which the SPH research community has turned its attention is the accuracy and order of convergence of the SPH scheme, also identified as one of the SPHERIC Grand Challenges [1]. In recent years, several approaches have been proposed to address this topic; in [2] and [3] a procedure based on the definition of several interpolation stencils for obtaining a WENO reconstruction, in the context of Riemann-SPH schemes, has been illustrated. Significant research effort has also focused on Eulerian SPH schemes, that are able to ensure a higher order of accuracy compared to the Lagrangian SPH formulation, due to the absence of the discretization error caused by the irregular distribution of particles [4]. Moreover, in the Eulerian SPH framework it is possible to obtain a convergence rate higher than the second order by means of high-order kernels that are obtained by relaxing the strictly positivity constraint, as demonstrated in [5]–[7].

In this work, a novel coupling between Lagrangian and Eulerian SPH schemes based on the variable resolution algorithm presented in [8] is proposed. The coupling is using a domain-decomposition strategy where the computational problem is partitioned in different sub-domains, for which a specific particle size is chosen. Additionally, in the present approach it is possible to define a specific formulation, i.e. Lagrangian or Eulerian and a different kernel (2nd or 4th order), to be used in the computational sub-domain. With this coupling, it is possible to combine the strength of both Lagrangian and Eulerian SPH schemes. The former is used in regions characterised by the presence of a free-surface interface or moving boundaries, while the latter, taking advantage of the better convergence rate due to high-order kernel functions, is more suited to discretised internal regions where complex vortex structures are generated and propagated.

The proposed algorithm is implemented in the DualSPHysics code framework.

The remainder of the manuscript is structured as follows: In Section II a brief summary of the numerical scheme and the coupling algorithm is presented, while in Section III the results for the flow past an impulsively started cylinder at a Reynolds number $Re = 3000$ are reported. Finally, in Section IV conclusions are drawn.

II. Coupling algorithm

The coupling algorithm herein proposed is based on the domain-decomposition technique used in the variable resolution approach in [8]. The key aspects of the algorithm are the following:

- The computational domain Γ is divided into a set of N subdomains Γ_i. For each subdomain, a distinct particle size dp_i, smoothing length h_i is used.
- Moreover, for each sub-domain specified, the kinematic description of the governing equations can either be Lagrangian or Eulerian, and in case of the latter, to the order of accuracy of the smoothing kernel function.
- The computational subproblems are closed by regions populated by particles that obtain their physical quantities by means of a corrected SPH interpolation [9] over the coupled subdomains.
- In case the Lagrangian formulation is used, as in [8], the generation of new particles in the sub-domain is based on the value of the Eulerian mass flux accumulated at specific points along the interface between coupled regions. In case the Eulerian description is employed, this step is ignored.

Thus, the proposed coupling allows for h/p refinement in SPH based on domain decomposition by choosing the appropriate description of motion and kernel weighting function. To the authors' knowledge, this is the first time such an approach has been demonstrated in SPH. For the 2nd order scheme, the Wendland kernel, W_{2C2}, [10] is chosen:

$$W_{2C2} = (1 - \frac{q}{2})^4(1 + 2q) \tag{1}$$

Following [5], the 4th order kernel schemes is obtained starting from the W_{2C6} kernel [10]:

$$W_{2C6} = (1 - \frac{q}{2})^8(4q^3 + 6.25^2 + 4q + 1). \tag{2}$$

The 4th order kernel has the form:

$$W_{4C6} = (A + Bq^2)W_{2C6}. \tag{3}$$

To find the constants A, B, the following conditions must be imposed:

$$\int_\Omega W d\Omega = 1, \tag{4}$$

$$\int_\Omega W q^2 d\Omega = 0. \tag{5}$$

where $q = r/h$. Please note that, in principle, higher order kernels can be adopted just imposing higher order moments constraints.

III. Results and Discussion

To validate the proposed approach, the flow past an impulsively started cylinder at a $Re = 3000$ is simulated. This particular flow is characterized by a complex interaction between primary and secondary vortex and, as demonstrated in [8], can only be accurately resolved if the boundary layer is discretized using a sufficiently fine particle size. For this reason, the refinement strategy consists of nesting progressively finer regions in a Russian doll-like manner.

The computational setup is similar to the one used in [8], where no-slip solid boundary conditions are imposed on the cylinder, while free-slip conditions are defined at the bottom and upper walls. Inflow-outflow boundary conditions are imposed using the approach presented in [11], and the velocity and density of the fluid region are initialized using a potential flow solution. The viscous stresses are discretized with the model presented in [12], and, to smooth the density field, the density diffusion term (DDT) proposed in [13] is used. The smoothing length-to-particle size ratio is selected equal to $h/\Delta x = 2.5$ for all regions, where h is the smoothing length and Δx is the initial inter-particle distance.

Three configurations were tested in this study: the first used a Lagrangian formulation across all subdomains. In contrast, the second and third configurations applied an Eulerian-SPH model to the inner regions, differing only in the order of the smoothing kernel function: second-order for the second configuration and fourth-order for the third.

To compare the accuracy of the different set-ups, a convergence study was performed by changing the maximum resolution near the cylinder, ranging from $D/\Delta x = 200$ to $D/\Delta x = 800$: in Figure 1a it can be seen that the Lagrangian SPH formulation with a resolution of $D/\delta x = 200$ is unable to correctly predict the history of the drag coefficient after the formation of the secondary vortex at a time $t^* = tU/D > 2$, losing the symmetry of the flow and starting to transition to a oscillatory solution. When the resolution is increased, the solution converges to the reference results, although with $D/\Delta x = 400$, a significant overprediction is still visible in the peak value of the drag coefficient at $t^* \approx 4$. When the 2nd order Eulerian SPH scheme is used, an overall improvement in the accuracy can be noticed in Figure 1b for all three different resolutions. For example, with a particle size equal to $D/\Delta x = 200$, the solution remains stable, although the flattening of the drag coefficient between t=2 and t=3 is poorly captured. Conversely, when the Eulerian SPH scheme is equipped with a 4th-order smoothing kernel, even at the lowest resolution, the numerical solution accurately describes the complex interaction between secondary and tertiary vortex (not shown here due to the lack of space). The comparison with the previous results reveals that the solution obtained with a 4th order kernel and $D/\Delta x = 200$ has comparable accuracy to the Lagrangian simulation using a resolution equal to $D/\Delta x = 400$. Moreover, Figure 1c shows also that for $D/\Delta x = 400$ the solution has converged to the reference results and no additional improvements are observed by further increasing the resolution.

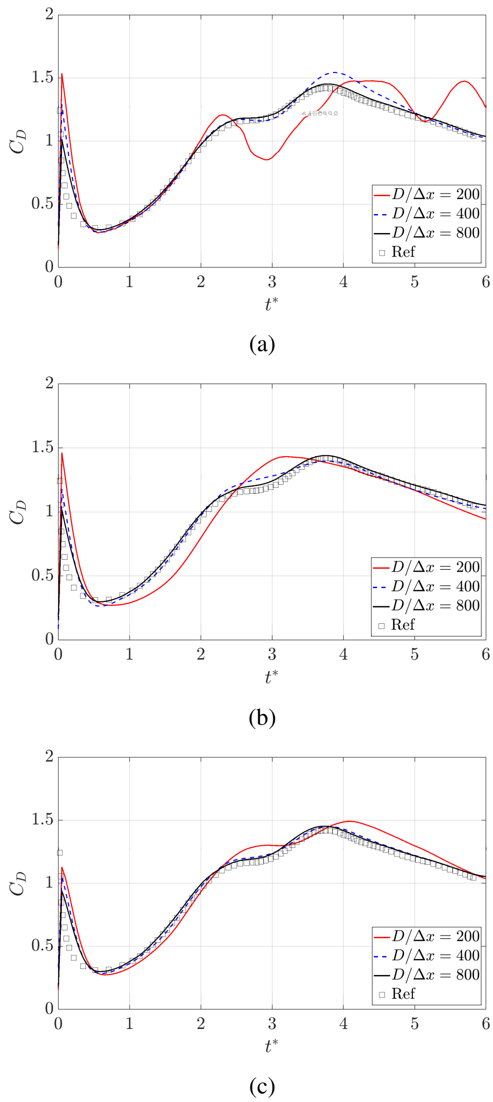
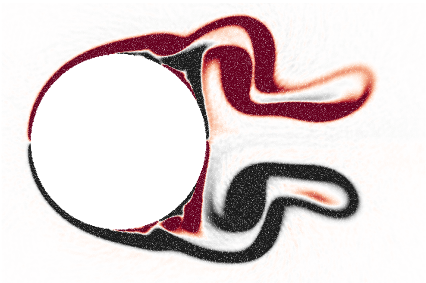
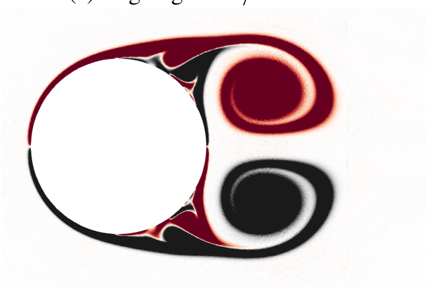


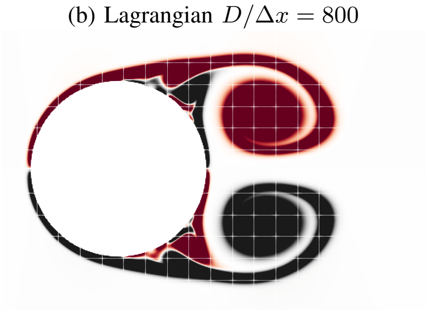
Fig. 1: Time history of the drag coefficient for the (a) Lagrangian and Eulerian-SPH with (b) 2nd and (c) 4th order schemes for flow past an impulsively started cylinder at a $Re = 3000$. Numerical results are compared to the reference solution in [14].



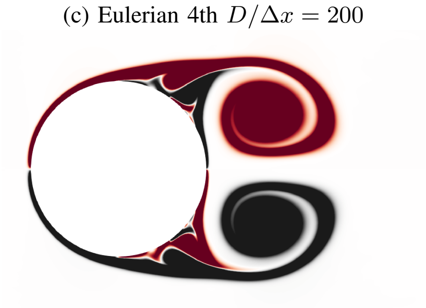
(a) Lagrangian $D/\Delta x = 200$



(b) Lagrangian $D/\Delta x = 800$



(c) Eulerian 4th $D/\Delta x = 200$



(d) Eulerian 4th $D/\Delta x = 800$

Fig. 2: Vorticity contours at time $t^* = 6$ for the flow past an impulsively started cylinder at $Re = 3000$.

To further investigate the results, in Figure 2 the vorticity contours are shown at $t^* = 6$. The solution obtained with a Lagrangian formulation at a resolution $D/\Delta x = 200$ shows a significant difference with the converged results when the particle size is reduced to $D/\Delta x = 800$, while the Eulerian results display a better rate of convergence.

IV. Conclusions

This work presents a novel Lagrangian-Eulerian SPH coupling methodology with h/p refinement. The algorithm is based on the domain-decomposition strategy proposed by Ricci et al. [2024] in [8] to introduce a variable resolution model that enables the application of Lagrangian or Eulerian SPH schemes in different regions of the computational domain. This approach allows the use of the Lagrangian formulation in regions involving complex interfaces, while leveraging the accuracy of the Eulerian fourth-order scheme in internal regions ensuring an accurate solution even in the presence of complex vortex structures. To validate this approach, the flow around an impulsively started cylinder at $Re = 3000$ was simulated. A comparison of the different configurations in the simulated test case demonstrated that the proposed algorithm can improve the order of accuracy of the numerical solution. Further test cases have not been included here due to the lack of space but they will be shown at the conference.

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