Towards High-Fidelity Simulation of Compressible Flows Using Adaptive Resolution SPH

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***Abstract* - This paper investigates adding adaptive resolution capabilities to compressible Smoothed Particle Hydrodynamics (SPH) to simulate high speed flows with shocks. A volume based adaptive refinement and de-refinement procedure is introduced to control the number density of particles in regions of interest. State-of-the-art features like automatic local adaptivity and solution adaptivity are explored in the context of shocks and compressible flows around bodies. This includes accurate handling of solid walls which is extremely important for industrial applications. The results demonstrate significant upsides and potential for further exploration. Overall, the study shows that adaptive resolution SPH can be a powerful tool for high-fidelity simulations of compressible flows with shocks.**

I. INTRODUCTION

In SPH, the number density of particles in a region can be controlled in-simulation with an adaptive refinement and derefinement procedure. An obvious application of this would be to resolve shocks better. Additionally, compressible flows can also result in scenarios where controlling the number of particles becomes more of a necessity than a convenience. For example, a low-density wake region is formed behind blunt bodies in compressible flows. As particle density is related to fluid density, the features in the wake region are poorly resolved. Moreover, particle volume can change significantly in compressible flows but keeping the particle volume in check is instrumental in controlling errors due to particle inconsistency.

II. BASE SCHEME AND ADAPTIVITY PROCEDURE

Literature that explores adaptive resolution SPH for compressible flows is sparse. Reference [2] is a study that addresses the need for adaptive resolution in compressible flows to some extent. However, this study is limited in the sense that it aims to use adaptive particle refinement merely as a means to impose bounds on the particle volumes, attempting to chase homogeneous and isotropic particle distributions. This does not explore the full potential of adaptive resolution SPH in compressible flows. Moreover, [2] makes use of a discretized form of the continuity equation to evolve density. We have found that this leads to post-shock density overestimation and incorrect shock speeds. We also observe that these issues are exacerbated in the presence of strong shocks. Furthermore, we suspect the use of a non-conservative discretization for continuity equation to be the cause of this issue and also find that is not possible to make this discretization conservative and consistent at the same time.

To this end, we use our implementation of the Matrix Inversion formulation 1 (MI1) scheme from MAGMA2 [5] as the base for adaptive refinement and derefinement procedure. We do not re-describe the base scheme here and refer readers to [5] for the details. We keep our implementation close to the MI1 scheme with just one modification. For pragmatic reasons, we swap out the smoothing length update procedure from MI1 in favour of summation density with iterative solution for the smoothing length procedure based on number density, from [6].

To drive refinement and de-refinement, we borrow the idea of volume based adaptivity from [2] and implement it within the parallelisation-friendly framework of [1]. We augment the capabilities of the adaptive resolution framework with automatic local adaptivity and solution adaptivity using ideas from [3] and [4]. We modify the solution adaptivity to track shocks and refine the resolution in the vicinity of shocks. We also introduce a shock limited shifting to regularize the particle distribution without compromising shocks.

For the details about refinement and derefinement, we refer the readers to [2]. In a nutshell, the volume of a particle exceeds a certain threshold, it is split into four equal volume particles, and if the volume is below a certain threshold, it is merged with its nearest neighbour that also satisfies the same condition. These upper and lower thresholds are derived from a reference volume, V_{ref} as $8V_{\text{ref}}/5$ and $2V_{\text{ref}}/3$, respectively. The reference volume itself is derived from local reference spacing, Δs. This is defined for each particle.

Let $\Delta s_{\max}$ and $\Delta s_{\min}$ be constants that denote the maximum and minimum reference spacing in the domain, respectively. Particles are initialized with $\Delta s_i = \Delta s_{\max}$, where i denote the particle index. For local adaptivity, Δs can be set as $\Delta s_{\min}$ if the particle is in a region of interest, say in the vicinity of a body or a shock. The technique of [3] and [4] is employed to increment Δs from the minimum value to the maximum value in steps of C_r; i.e., if Δs_k is the reference spacing for k^{th} step, the reference spacing for $(\text{k} + 1)^{\text{th}}$ step would be $\Delta s_{k+1} = C_r \Delta s_k$.

A. *Shock based solution adaptivity*

The presence of a negative divergence of velocity is exploited to identify particles near shocked regions. The minimum divergence in the domain is used to normalize the negative divergence as

$$\varsigma_i = \frac{\langle \nabla \cdot \mathbf{u} \rangle_i}{\min_i \left(\langle \nabla \cdot \mathbf{u} \rangle_i \right)}, \tag{1}$$

where $\mathbf{u}$ is the velocity and the angle brackets denote the SPH approximation of the operation. With ς_i, we can identify a band or particles particles in the vicinity of a shock. As this band is very narrow, it is widened by introducing ς_s, the maximum ς from among the neighbours of a particle as

$$\varsigma_{s,i} = \max_j (\varsigma_j). \tag{2}$$

The reference spacing can be assigned as

$$\Delta s_i = \Delta s_{\min} \text{ if } \varsigma_{s,i} > \varsigma_{rst}, \tag{3}$$

where ς_{rst} is the threshold set as 0.6.

B. *Shock limited shifting*

The shifting of particles is a common technique to regularize the particle distribution. However, shifting is not conservative and is undesirable near shocks. We introduce a shock limited shifting based on [7] to regularize the particle distribution without compromising shocks.

$$\delta \mathbf{u}_i = 0.5 \begin{cases} 0 & \text{if } \varsigma_{s,i} < \varsigma_{rst}, \\ c_f \left(\frac{R}{\Delta x}\right)^3 R \widehat{\nabla} C_i & \text{if } \left\| \left(\frac{R}{\Delta x}\right)^3 R \widehat{\nabla} C_i \right\| < \frac{1}{2} \frac{R}{\Delta x} \\ c_f \frac{1}{2} \frac{R}{\Delta x} \frac{\widehat{\nabla} C_i}{\|\widehat{\nabla} C_i\|} & \text{otherwise,} \end{cases} \tag{4}$$

where

$$c_f = - \max_{j \in \mathcal{N}} \left(\left| (\mathbf{u}_j - \mathbf{u}_i) \cdot \frac{\mathbf{r}_j - \mathbf{r}_i}{\|\mathbf{r}_j - \mathbf{r}_i\|} \right| \right) \tag{5}$$

and

$$\widehat{\nabla} C_i = \sum_{j \in \mathcal{N}} \left[1 + 0.2 \left(\frac{W_{ij}}{W(\Delta x)} \right)^4 \right] \frac{m_j}{\rho_j} \nabla W_{ij}. \tag{6}$$

Here, W is the kernel, R is the kernel radius, $\mathbf{r}$ is the position vector, h is the smoothing length, Δx is taken as the smoothing length times the inflection point of the kernel.

This combination allows us to simulate various compressible flow scenarios with shocks and bodies. Some results are shown in the next section.

C. *Boundary treatment*

We use the boundary treatment method from [8] with additional considerations for compressible flows prescribed by [9]. This method involves use of ghost particles. Properties like e, p and h are extrapolated from the fluid particles to the ghost particles. The ghost particle velocities are then flipped about the tangent to the interface while keeping their volume unchanged. The density of the ghost particles is obtained using the extrapolated e and computed p in the equation of state. The mass m of the ghost particles is modified to make sure that the representative volume, m/ρ, remains constant. If a fluid particle is on course to penetrate the interface, it is steered away by the shield making use transport velocity as described in [9]

III. RESULTS

Figure 1 from [9] highlights the issues with the double Mach reflection problem when run without adaptive resolution. These instabilities lead to blow-up at finer resolutions. Figure 2 shows that clumping and kinked Mach stem are resolved just by using simple volume adaptivity from [2], without even using shifting or solution adaptivity. The post shock density overshoot was resolved by changing the scheme to MI1. Being able to run at finer resolutions also allows to observe the Kelvin-Helmholtz instabilities about the slip line.

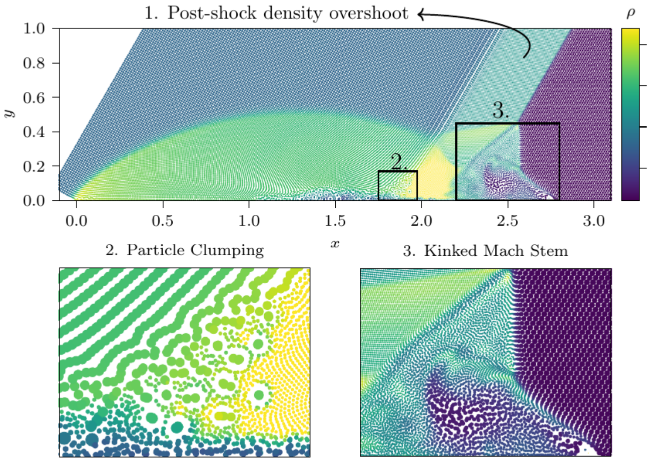

Fig. 1. Double mach reflection without adaptive refinement from [9]

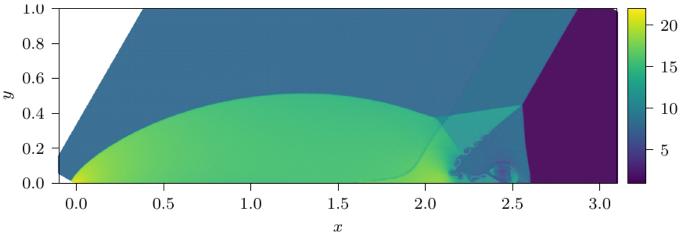

Fig. 2. Double mach reflection with adaptive refinement

Figure 3 shows the divergence of velocity for a compression corner problem highlighting the effectiveness of solution adaptivity in sharper resolution of shocks.

Figure 4 shows the Mach number around a biconvex airfoil in a supersonic flow. The pressure distribution in shown in fig. 5. This result demonstrates how refining the resolution in the vicinity of the airfoil and solution adaptivity in the vicinity of the shock makes the pressure distribution over the airfoil better.

IV. CONCLUSIONS

We have highlighted the importance of choosing the right discretization scheme when shocks are involved. We have also

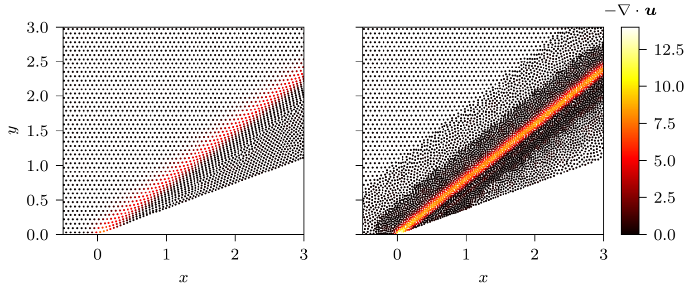

Fig. 3. Compression corner with adaptive refinement

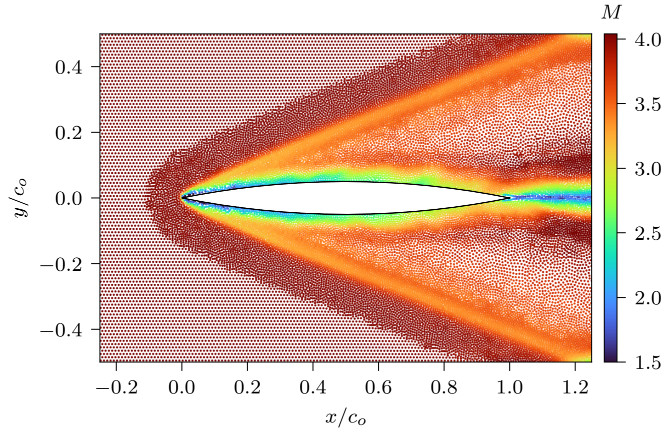

Fig. 4. Mach number distribution over biconvex airfoil in supersonic flow

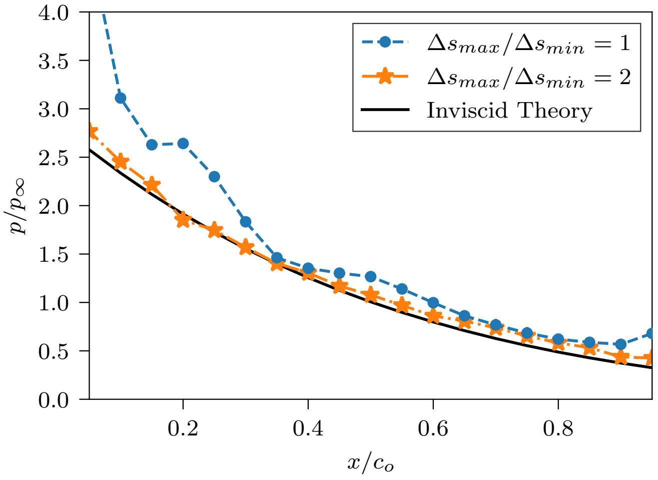

Fig. 5. Pressure distribution over biconvex airfoil in supersonic flow

shown how adaptive resolution can significantly enhance the results of compressible SPH simulations that involve flow in and around bodies. The use of solution adaptivity for better resolution of shocks has also been demonstrated. Though it has not been shown here, the presented methods also work well in 3D problems. As we continue to work on some more algorithmic improvements and optimizations these promising results drive the point that adaptive resolution SPH can be a powerful tool for high-fidelity simulations of compressible flows with shocks.

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