Developments in meshfree modeling for multiphysics laboratory experiments, astrophysics, and planetary defense

Presented at the 2025 SPHERIC World Conference

June 17-19, 2025

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Prepared by LLNL under Contract DE-AC52-07NA27344.

Meshfree methods have been around for decades, but acceptance is spotty across different fields.

- Smoothed Particle Hydrodynamics (SPH[1,2]) is the ancestor of most of the more general meshfree methods we use today.
- Those early papers from the 70's and 80's developed SPH to provide hydrodynamics in N-body gravitational models, where meshfree is the most natural discretization.
- The Lagrangian nature of SPH was also an essential practicality.
  - Gravitationally driven collapse leads to large changes in the required resolution (orders of magnitude), and this was problematic for the standard meshed hydrodynamics to resolve on all scales/times.

[1]Lucy, L.B., 1977. A numerical approach to the testing of the fission hypothesis. Astron. J. 82, 1013–1024.
[2]Gingold, R.A., Monaghan, J.J., 1977. Smoothed particle hydrodynamics: theory and application to non-spherical stars. Mon. Not. R. Astron. Soc. 181, 375–389.

The early experiments with SPH were rather successful, but why?

- SPH (particularly early on) was quite a bit less sophisticated than the competing meshed based hydrodynamics algorithms of the time.
  - SPH interpolation is quite crude, with many deficiencies we are all too aware of:
    - Lack of reproducibility for fields of any order – for points that are are not perfectly uniform SPH is not even zeroth-order consistent!
    - Even granted this lack of reproducibility SPH interpolation also tends to be noisy.
      - Some folks analyzed SPH interpolation as a Monte-Carlo approximation, but it's not that bad!
  - Adding more neighbors reduces the noise but aggravates the fact that SPH also tends to be more expensive than competing meshed discretizations (on a cost per element basis, which is not necessarily entirely fair depending on the problem).
- One important factor is that SPH (and related methods) can be formulated to match the relevant physics invariants for many classes of problems.

The traditional SPH equations preserve many of the physics invariants important in their applications.

- Despite its many numerical drawbacks, even the early traditional SPH formulations preserve a remarkable set of important physical principles:
  - Exact conservation of mass, linear, and angular momentum
    - Even better these properties are expressed on a local pairwise basis
  - Galilean invariance (i.e., the answer is frame independent)
  - The numerics are naturally expressed in a Lagrangian framework
    - Just as importantly the method remains robust in that moving framework
- Competing mesh based discretizations have difficulties with the highlighted properties, even to this day.
  - Despite the fact such methods were (and are) substantially more numerically sophisticated (better accuracy, less diffusive, much more amenable to analysis), they often struggle in problem areas where SPH succeeds.

This principle of reflecting the physical properties of the continuum equations is well-known.

- The idea of exactly reproducing properties of continuum equations in their discrete counterparts has been referred to as "mimetic" [1].
  - This just means the discrete equations "mimic" (or reproduce) some set of crucial properties from the continuum physics equations.
  - Note: "exact" discrete conservation means to numerical roundoff.
  - These ideas go back to at least the 1960's
- Related literature[2] also refers to these discretization as "compatible".
- Examples from the broader world of numerical modeling include:
  - Exact conservation of mass, momentum, and energy
  - Exact divergence-free velocity fields for incompressible flow
  - Exact divergence-free magnetic fields for ideal magneto-hydrodynamics

[1] Lipnikov, K., Manzini, G., Shashkov, M., 2014. Mimetic finite difference method. J. Comput. Phys. 257, 1163–1227. https://doi.org/10.1016/j.jcp.2013.07.031.
[2] Caramana, E.J., Burton, D.E., Shashkov, M.J., 1998. The construction of compatible hydrodynamics algorithms utilizing conservation of total energy. J. Comput. Phys. 146, 227–262.

Know your problem: numerical methods are a series of compromises – what's important for your case?

- Many of the problems I work with are dominated by the presence and evolution of strong shocks in compressible flows.
  - Astrophysics: supernova, large-scale cosmological structure, galaxy formation
  - Laboratory physics: Laser-driven experiments, inertial confinement fusion (ICF)
  - Planetary defense: cratering and impact physics on planetary bodies and asteroids
- All the conservation properties SPH already meets mentioned previously are important: mass, linear momentum, and angular momentum.
  - Also frame (Galilean) invariance and working in a Lagrangian frame.
- Two more properties are critical however: energy conservation and entropy preservation.

Exact energy conservation is achievable, but how we do it matters!

- Many non-conservative formulations evolve the specific thermal energy (*u*) via a PDE representation for $\frac{Du}{Dt} = \sigma^{\alpha\beta}\dot{\epsilon}^{\beta\alpha}, \quad \dot{\epsilon}^{\alpha\beta} = \frac{1}{2}\left(\partial_\beta v^\alpha + \partial_\alpha v^\beta\right)$
  - This approach generally yields reasonable entropy evolution but does not conserve total energy

- It is simple to exactly conserve energy by directly evolving the total energy.
  - This is very common in Eulerian discretizations for strong shocks
  - While energy is conserved to roundoff, there are serious drawbacks to this approach
    - The thermal energy must be formed from the difference $u = \frac{E}{m} - \frac{1}{2}v^2$
    - This sort of difference can lead to noisy thermal energy, temperature, and poor entropy evolution

- It is also possible to directly evolve the entropy[1] rather than energy.
  - This yields good entropy and thermal energy behavior, but does not conserve energy
  - Also can be difficult to work with general equations of state

[1]Springel, V., Hernquist, L., 2002. Cosmological smoothed particle hydrodynamics simulations: the entropy equation. Mon. Not. Roy. Astron. Soc. 333, 649. https://doi.org/10.1046/j.1365-8711.2002.05445.x

We can construct an exactly energy conserving scheme "compatibly" with the momentum discretization[1].

- The momentum equation we solve for the velocity update is generally constructed to enforce exact linear momentum conservation.
- Starting from the total energy conservation equation, we can account for the exact discrete work from the velocity update.

$$E^1 - E^0 = \sum_i m_i \left[\frac{1}{2} \left((v_i^1)^2 - (v_i^0)^2 \right) + u_i^1 - u_i^0 \right] = 0$$

$$\Delta E_{ij}^{\text{thermal}} = m_i \left[(v_j^\alpha)^{1/2} - (v_i^\alpha)^{1/2} \right] (a_{ij}^\alpha)^0 \Delta t$$

$$\Delta u_{ij} = f_{ij} \Delta E_{ij}^{\text{thermal}} / m_i$$

- f_{ij} is the fraction of the pairwise work applied to point *i* of pair (*i*,*j*)
- So long as $f_{ij} + f_{ji} = 1$, exact energy conservation is maintained.

[1]Owen, J.M., 2014. A compatibly differenced total energy conserving form of SPH. International Journal for numerical methods in fluids 75, 749–774. https://doi.org/10.1002/fld.3912

The physics in the compatible energy update is now in choosing the conservative partitioning in f_{ij}

- In our original paper we show it is possible to reproduce conservative forms of several classic SPH energy equations for choices of f_{ij} (such as ½).
- However, we are free to choose this partitioning depending on our goal:
  - Variation diminishing in thermal energy (smoothly or monotonically)
  - Equipartitioning of entropy changes
- In general entropy is treated at least as accurately as the non-conservative approaches.
- Perhaps just as importantly, this formalism works for any discretization producing a discrete pairwise acceleration.
  - We use this compatible energy update for all our meshfree discretizations (both fluid and solid): SPH, CRKSPH, FSISPH, SVPH, …

Conservative Reproducing Kernel Hydrodynamics

Including work with Cody Raskin, Nicholas Frontiere, and Will Gray

Reproducing Kernels: one approach to improving the underlying numerics of our methods.

- As we mentioned earlier, SPH is not a great interpolation method due low accuracy and lack of consistency.

- Reproducing Kernels (RK)[1,2] proposes enriching the interpolation basis in order to enforce accuracy and consistency:

$$W(x_{ij}^{\alpha}, h) \rightarrow W^R(x_{ij}^{\alpha}, h) = A_i \left(1 + B_i^{\alpha} x_{ij}^{\alpha} + \ldots\right) W(x_{ij}^{\alpha}, h)$$

  - This form corrects to linear terms, so linear fields are reproduced exactly.

- While Reproducing Kernels are a huge improvement in accuracy, the simplest formulations give up momentum conservation (as well as energy).
  - Due to the fact that $W^R(x_{ij}^{\alpha}, h)$ is not symmetric in (*i,j*)
  - Acceptable for the low-energy engineering applications RK was developed for, but entirely inappropriate for the high-energy strong shock regime.

[1] Liu, W.K., Jun, S., Zhang, Y.F., 1995. Reproducing kernel particle methods. International Journal for numerical methods in fluids 20, 1081–1106.
[2] Bonet, J., Kulasegaram, S., 2000. Correction and stabilization of smooth particle hydrodynamics IJNME, V 47, Issue 6, 1189-1214.

We can derive a form of Reproducing Kernels that is conservative for our applications.

- The loss of symmetry in the reproducing kernel complicates enforcing exact momentum conservation, but it is still possible.
- We derive and extensively test a Conservative Reproducing Kernel (CRKSPH)[1] formulation with a momentum equation of the form

$$m_i \frac{Dv_i^\alpha}{Dt} = -\frac{1}{2} \sum_j V_i V_j (P_i + P_j) \left(\partial_\alpha W_{ij}^R - \partial_\alpha W_{ji}^R\right)$$

- CRKSPH exactly conserves mass, momentum, and energy (via the compatible energy methodology already described).
  - Improvements such as a monotonically limited artificial viscosity are also introduced in this paper.

[1] Frontiere, N., Raskin, C.D., Owen, J.M., 2017. CRKSPH – A Conservative Reproducing Kernel Smoothed Particle Hydrodynamics Scheme. Journal of Computational Physics 332, 160–209.

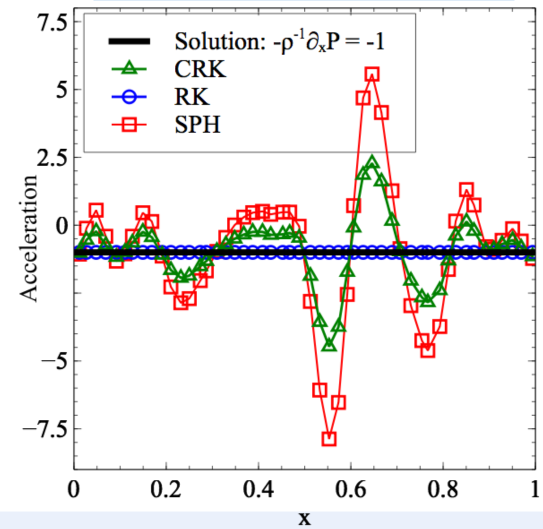
CRKSPH represents a tradeoff between consistency and conservation.

- Note the CRKSPH momentum equation implies we are sacrificing the exact consistency of gradients gained by a direct RK implementation.
  - CRKSPH (conservative): $$m_i \frac{Dv_i^\alpha}{Dt} = -\frac{1}{2} \sum_j V_i V_j (P_i + P_j) \left(\partial_\alpha W_{ij}^R - \partial_\alpha W_{ji}^R \right)$$
  - RK (consistent): $$m_i \frac{Dv_i^\alpha}{Dt} = -\sum_j V_j P_j \partial_\alpha W_{ij}^R$$
- Evaluating the above relations on an analytic pressure distribution demonstrates this tradeoff:

$$P(x) = 1.0 + x, \quad \rho(x) = 1$$

$$\frac{Dv}{Dx} = -\rho^{-1} \partial_\alpha P = -1$$

CRK vs RK acceleration for an analytic pressure distribution

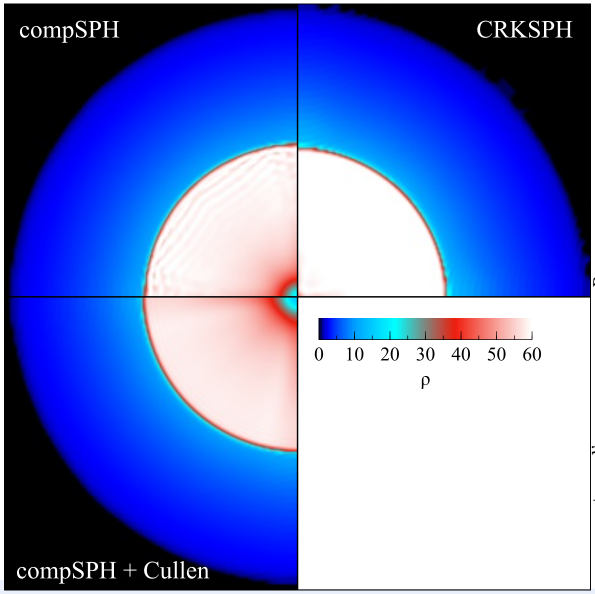


- We pay a price for conservation, but it's essential!

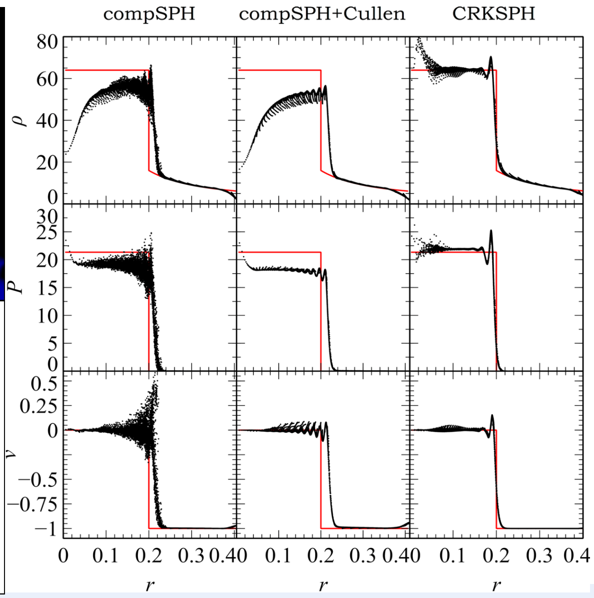
CRKSPH performs well across a wide variety of shock dominated compressible problems.

- CRKSPH demonstrates our analytically expected first-order convergence for shock dominated problems.
  - Also greater accuracy than the competing SPH methodologies.

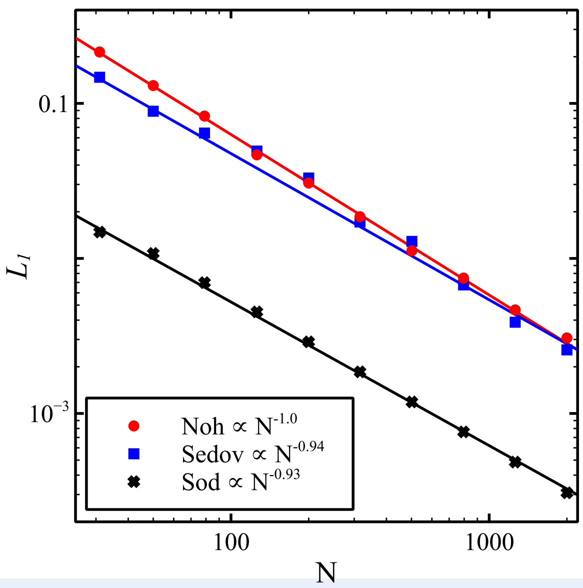
Slices through Noh (3D) implosion



Noh implosion (3D) profiles



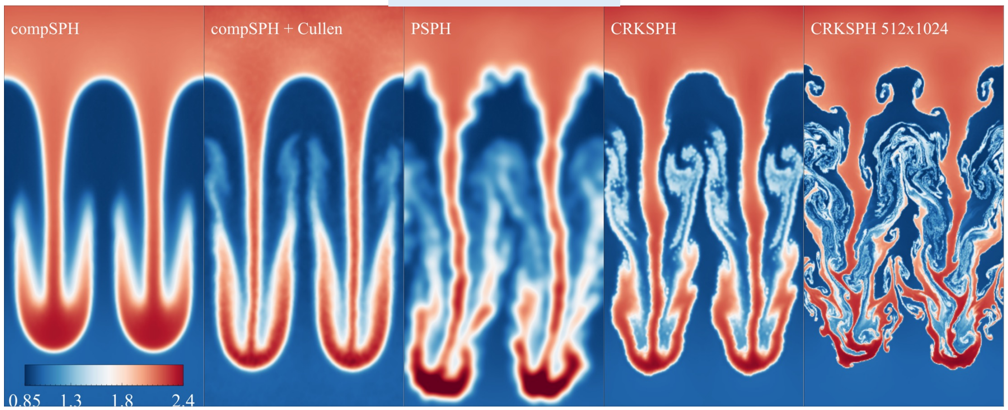
Convergence rates across shock problems



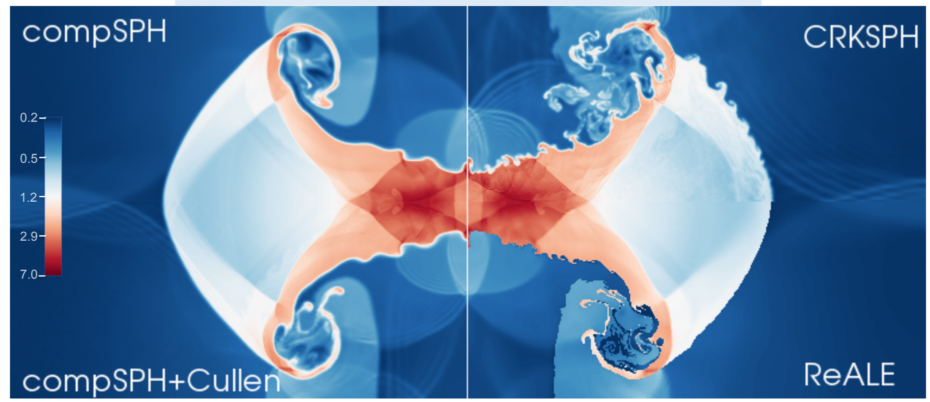
The improved accuracy of CRKSPH is important for modeling the evolution of fluid instabilities.

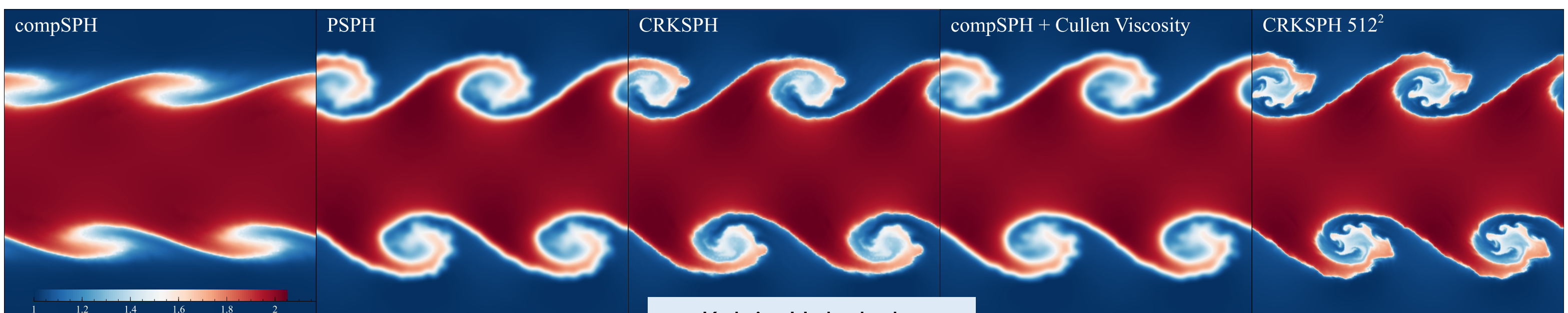
- Of particular importance is the reduction in numerical diffusion.

Rayleigh-Taylor



“Triple point” (3 region shock-tube)



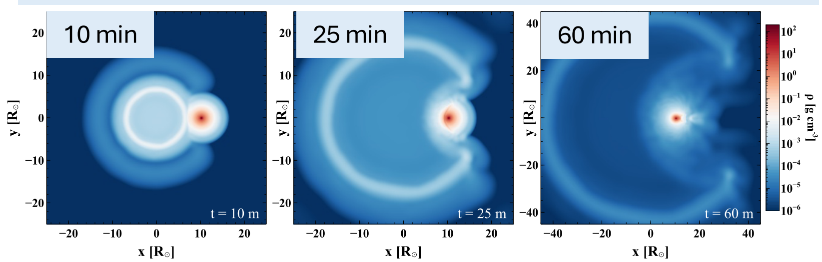


Kelvin-Helmholtz

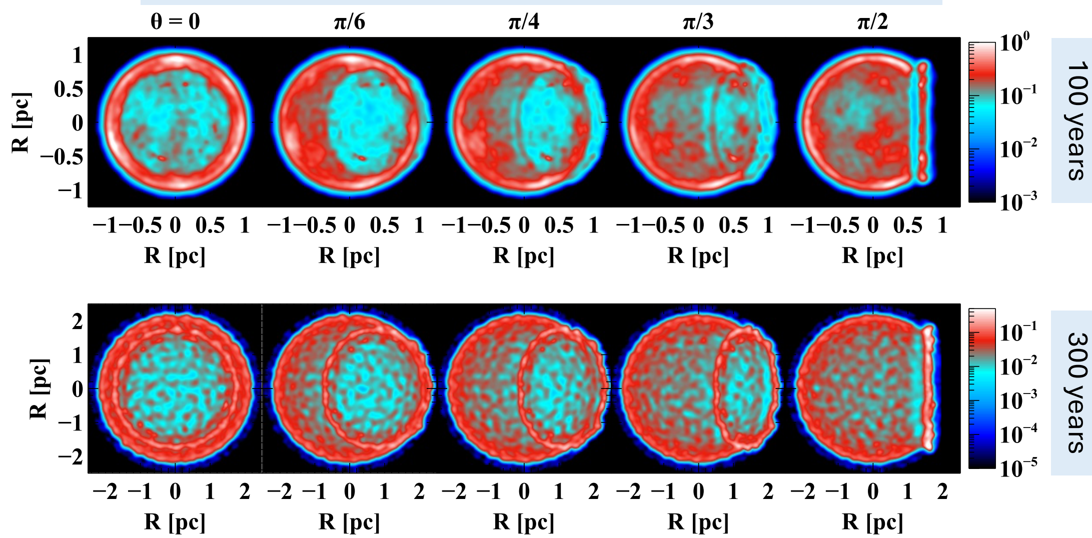
One application area we've explored with CRKSPH is supernova remnants.

- Testing classical theory for Type Ia supernova:
  - Gas transfer from main-sequence star to white dwarf – will the existence of the donor star be evident by a hole in the supernova remnant?

Time evolution of shock wrapping around primary star



Projected X-ray images from various viewing angles



Note: density variations in these images are due to Rayleigh-Taylor instabilities

- This analysis suggests we should see the final asymmetry in the SN remnant.
  - Not clear in any observed to date.
  - Support for white-dwarf collisions as Type 1a progenitors?

[1]Gray, W.J., Raskin, C., Owen, J.M., 2016. Shadows of our former companions: how the single-degenerate binary type ia supernova scenario affects remnants. Astrophys. J. 833, 62. https://doi.org/10.3847/1538-4357/833/1/62

Radiation Hydrodynamics

Work with Brody Bassett and Tom Brunner

Radiation hydrodynamics is of interest for a wide variety of laboratory and astrophysical problems.

- The diffusion rad-hydro conservation equations we need to solve are

$$\frac{D\rho}{Dt} = -\rho\partial_\alpha v^\alpha$$
$$\rho\frac{Dv^\alpha}{Dt} = -\partial_\alpha P - \lambda\partial_\alpha E_r$$
$$\rho\frac{Du}{Dt} = -\partial_\alpha P v^\alpha - c\sigma_a B + c\sigma_a E_r + Q_u$$
$$\frac{DE_r}{Dt} = -\frac{4}{3}E_r\partial_\alpha v^\alpha + \partial_\alpha \frac{c\lambda}{\sigma_t}\partial_\alpha E_r - c\sigma_a E_r + c\sigma_a B + Q_{E_r}$$
$$c^{-1}\frac{DF^\alpha}{Dt} = -c^{-1}F^\alpha\partial_\beta v^\beta - c\partial_\beta P_r^{\alpha\beta} - \sigma_t F^\alpha$$

E_r	radiation energy density
F^α	radiation flux
$P_r^{\alpha\beta}$	radiation pressure
λ	flux limiter
σ_a	absorption opacity
σ_t	total opacity
B	integrated photon emission
c	speed of light
Q_u	material energy source
Q_{E_r}	radiation energy source

- We have integrated over all radiation energies and angles from the full transport equations (i.e., “gray diffusion”)[1,2]

[1]Bassett, B.R., Owen, J.M., Brunner, T.A., 2021. Efficient smoothed particle radiation hydrodynamics I: thermal radiative transfer. J. Comput. Phys. 429, 109996.
[2]Bassett, B.R., Owen, J.M., Brunner, T.A., 2021. Efficient smoothed particle radiation hydrodynamics II: radiation hydrodynamics. J. Comput. Phys. 429, 109994.

We operator split the update into hydrodynamics, radiation, and radiative work.

- The radiation step uses implicit time integration (backward Euler).
  - We also need a second-derivative operator for the $\partial_\alpha \frac{c\lambda}{\sigma_t} \partial_\alpha E_r$ term.
  - The common SPH low-order discretization works surprisingly well

$$\left\langle \partial_\alpha \frac{c\lambda}{\sigma_t} \partial_\alpha E_r \right\rangle \approx \sum_j V_j \left(D_i + D_j\right) \left(E_{r\,i} - E_{r\,j}\right) \frac{x_{ij}^\alpha}{x_{ij}^\beta x_{ij}^\beta} \partial_\alpha W_{ij}, \quad D_i \equiv \frac{c\lambda_i}{\sigma_{t\,i}}$$

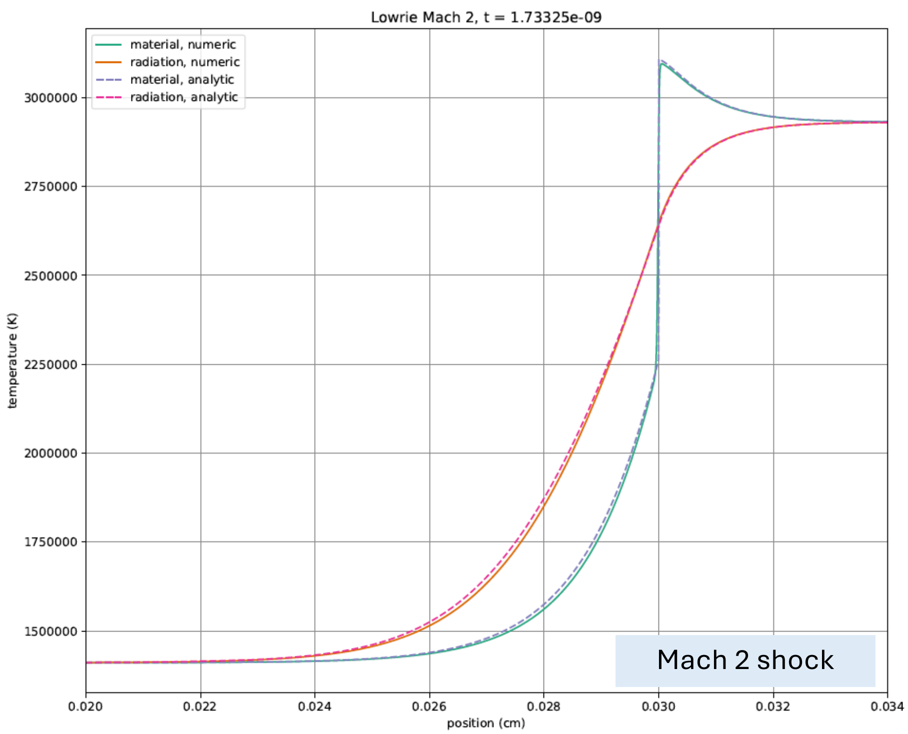
  - We also have a RK version of the diffusion operator that works well

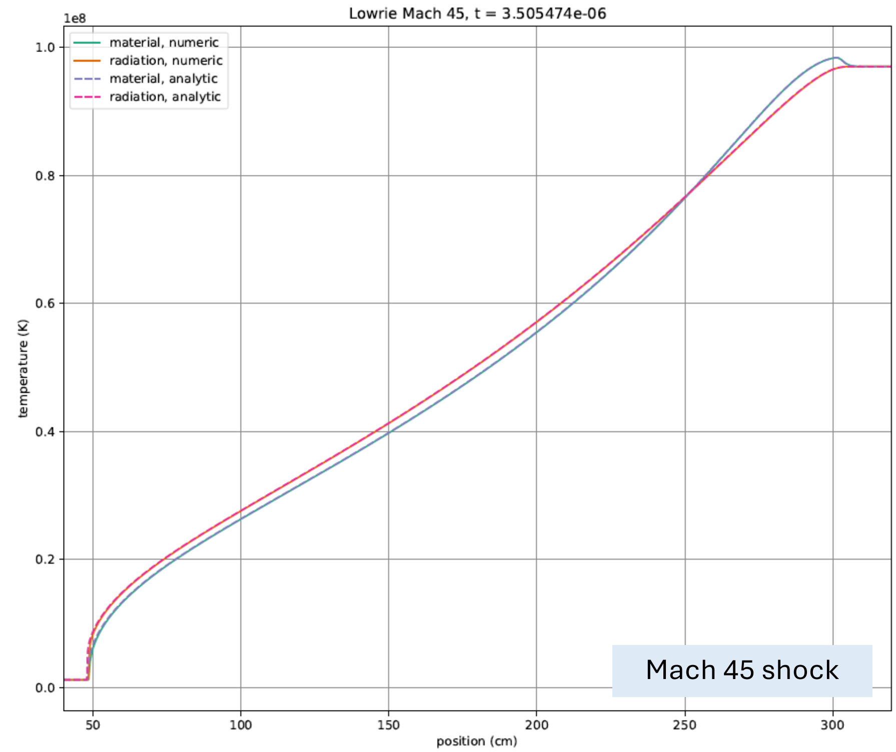
- The implicit time discretization is challenging due to the highly nonlinear terms involved.
  - Our work is based on the hard-won knowledge of experts in transport preceding us.
    - We are using the method of nonlinear elimination[1,2]
  - The details are too much to cover here, but see the refs by Brody (previous slide) and below.

[1] Brunner, T.A., Haut, T.S., Nowak, P.F., 2020. Nonlinear Elimination Applied to Radiation Diffusion. Nuclear Science and Engineering 1–13.
[2] Morel, J.E., Brian Yang, T.-Y., Warsa, J.S., 2007. Linear multifrequency-grey acceleration recast for preconditioned krylov iterations. JCP. 227, 244–263

The Lowrie radiating shock[1] is a semi-analytic radiation-hydrodynamics problem and provides an essential test.

- Rad-hydro problems with an analytic solution are scarce!

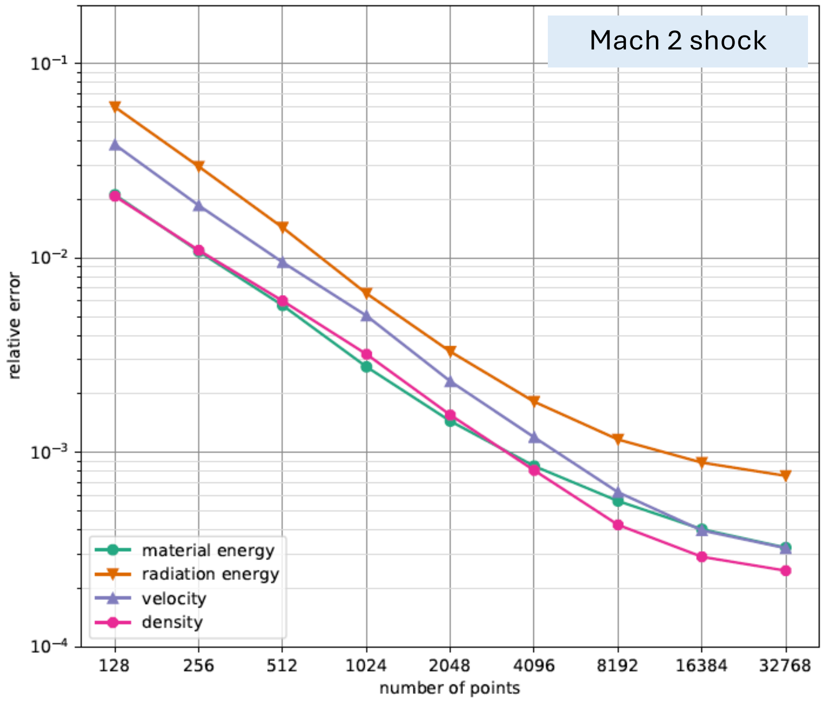


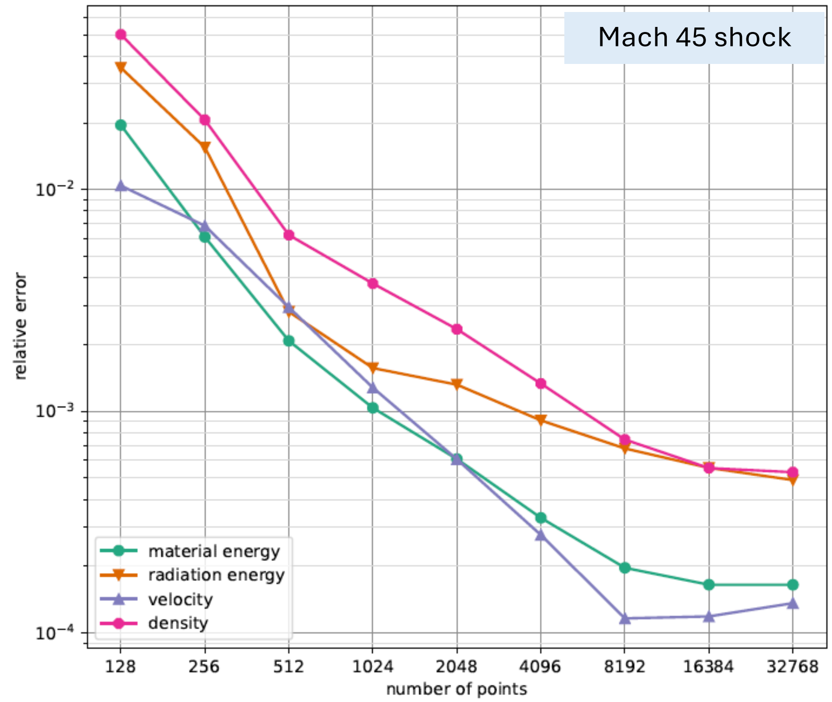


[1]Lowrie, R.B., Edwards, J.D., 2008. Radiative shock solutions with grey nonequilibrium diffusion. Shock Waves 18, 129–143. https://doi.org/10.1007/s00193-008-0143-0

The Lowrie radiating shock demonstrates second-order convergence (until limited by time step).

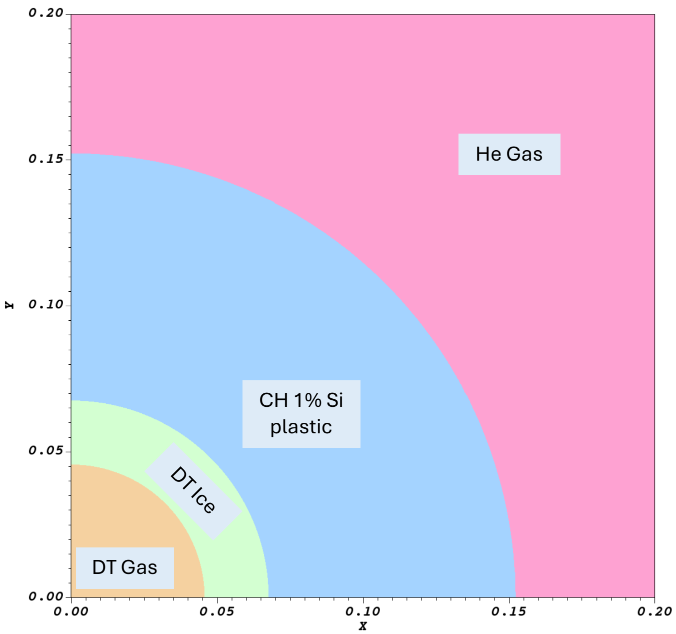
- Second-order is only possible here because radiative effects smooth the discontinuities.

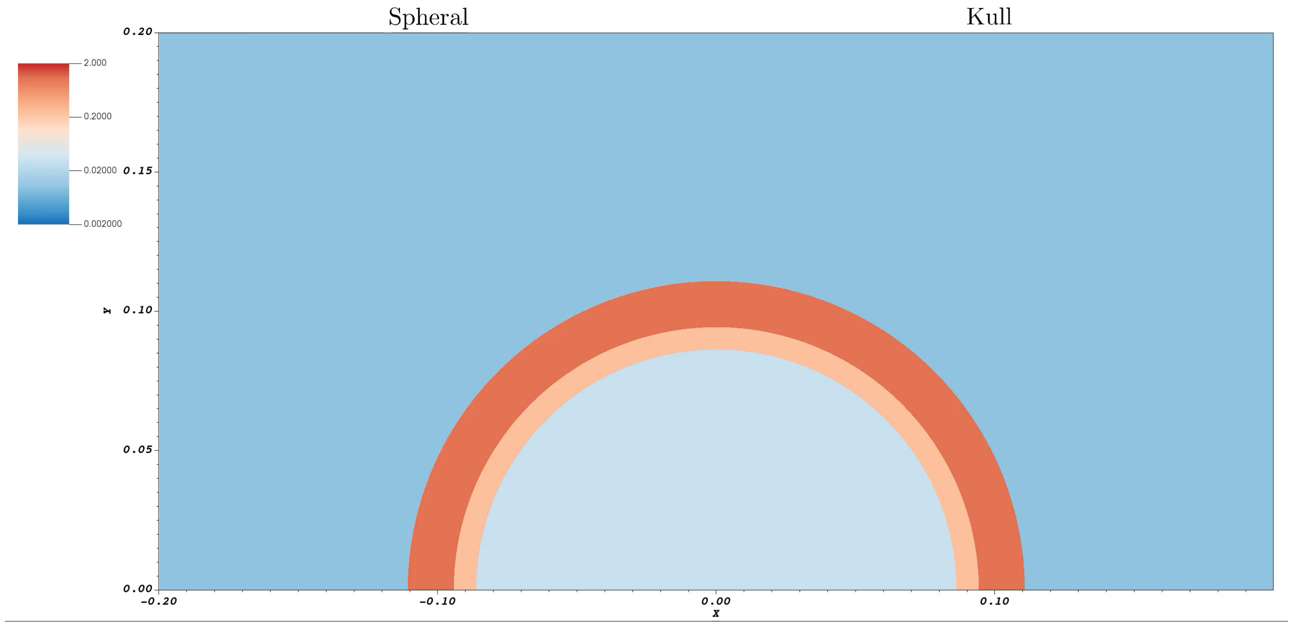




An idealized ICF-like ablation problem shows promise when comparing with more mature codes.

- This is a simplified example of a test case proposed by Robert Tipton[1].
  - Comparison with Kull shows reasonable agreement for very different methodologies.
  - For more details see Brody's paper [2].





[1]Robert Tipton, 2016. An ICF test problem for single group radiation-hydrodynamics codes (personal communication).
[2]Bassett, B.R., Owen, J.M., Brunner, T.A., 2021. Efficient smoothed particle radiation hydrodynamics II: radiation hydrodynamics. JCP. 429, 109994.

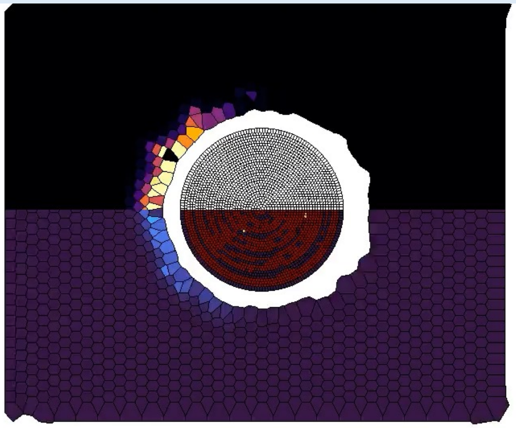
Fluid-Solid Interface Smoothed Particle Hydrodynamics

Work with Jason Pearl and Cody Raskin

Compressible flows with high-density ratio materials in contact requires some special treatment.

- Our target problem is following the entry, breakup, and energy deposition from small asteroids (or bolides) in Earth's atmosphere.
  - This problem entails high-Mach number flows (with entry velocities ~20 km/sec) and density contrasts ~10^4
  - Requires compressible and solid hydrodynamics, porosity, damage modeling, fracture and failure with strong shocks.
- The standard symmetrized momentum equation in SPH methods results in an artificial repulsion between points with differing masses.
  - In our case with mass ratios ~10^4 between air and rock nodes this error is unacceptable.

Standard SPH model of bolide in air



FSISPH[1] modifies the standard SPH equations explicitly to handle interfaces with high density contrasts.

- We bring together components of other discretizations combined with a few newish tidbits at interfaces.

$$\frac{D\rho_i}{Dt} = -2\rho_i \sum_j \frac{m_j}{\rho_j} \left(v_i^\alpha - v_*^\alpha\right) \partial_\alpha W_i$$

$$\frac{Dv_i^\alpha}{Dt} = \frac{m_j}{\rho_i \rho_j} \left(\sigma_i^{\alpha\beta} \partial_\beta W_i - \sigma_j^{\alpha\beta} \partial_\beta W_j\right)$$

$$\frac{Du_i}{Dt} = 2 \sum_j \frac{m_j}{\rho_i \rho_j} \left(\sigma_i^{\alpha\beta} \partial_\beta W_i (v_i^\alpha - v_*^\alpha)\right)$$

$$\rho_i = \sum_j m_* W_i$$

$$m_* = \begin{cases} m_j, & \text{same material} \\ m_i, & \text{different material} \end{cases}$$

$$\Pi_{ij} = \frac{\rho_i \rho_j}{\rho_i + \rho_j} C_l c_{ij} \mu_{ij} + C_q \mu_{ij}^2$$

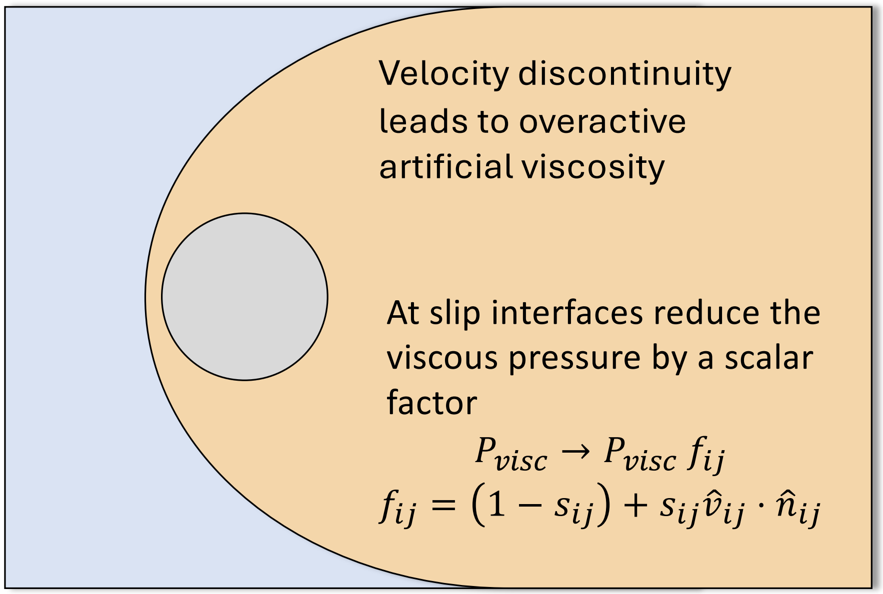
$$v^{\dagger\alpha}_{ij} = v_i^\alpha - v_j^\alpha - \frac{\phi_{ij}}{2} \left(\partial_\beta v_i^\alpha + \partial_\beta v_j^\alpha\right) x_{ij}^\alpha$$

$$\mu_{ij} = \max(-v^{\dagger\alpha}_{ij} \hat{x}_{ij}^\alpha,\ 0)$$

- GDF (geometric density averaged force)[2] helps with differing node masses
- HLLC reconstructed interface velocity at material boundaries[1]
- Second-order artificial viscosity improves shock capturing[3]

[1]Pearl, J.M., Raskin, C.D., Michael Owen, J., 2022. FSISPH: An SPH formulation for impacts between dissimilar materials. JCP 469, 111533.
[2]Wadsley, J.W., Keller, B.W., Quinn, T.R., 2017. Gasoline2: a modern smoothed particle hydrodynamics code. Mon. Not. R. Astron. Soc. 471, 2357.
[3]Frontiere, N., Raskin, C.D., Owen, J.M., 2017. CRKSPH – A Conservative Reproducing Kernel Smoothed Particle Hydrodynamics Scheme. JCP 332, 160–209

For the bolide problem we need to add explicit surface treatments at material interfaces.



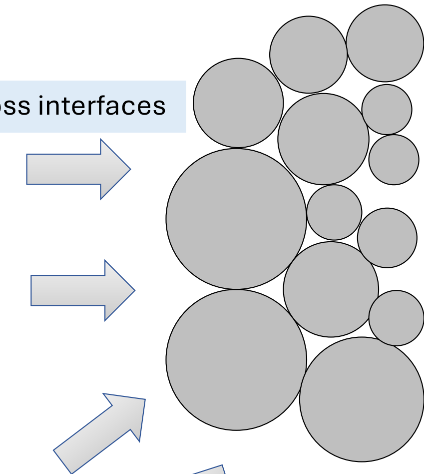
s_{ij} - surface smoothness ranges between 0 and 1 used to turn off slide surface when interface and normals are poorly defined.

$\hat{n}_{ij}$ - orientation of surface normal

In compression materials couple across interfaces

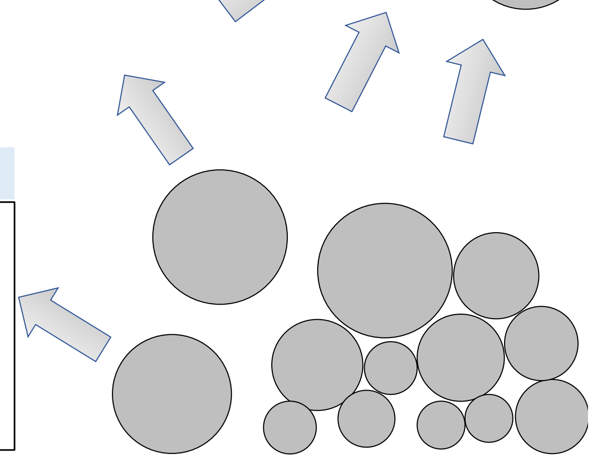
Damaged rubble and material interfaces can support compression but separate in tension.

After separation (P<0) density and energy should no longer evolve

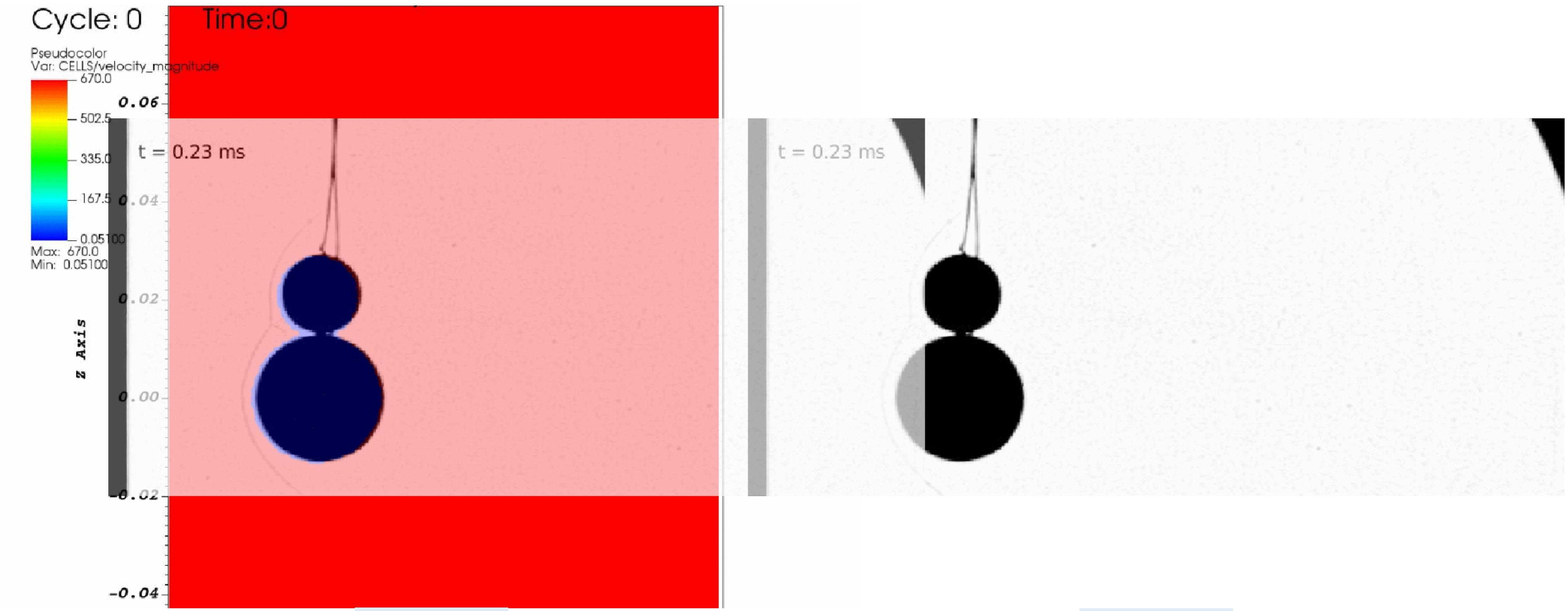


Interfaces decouple if:

- Pairwise tensile force
- Particles separating
- Different materials or pairwise interaction is fully damaged



FSISPH performs well modeling the separation of spheres in experimental wind tunnel tests[1].



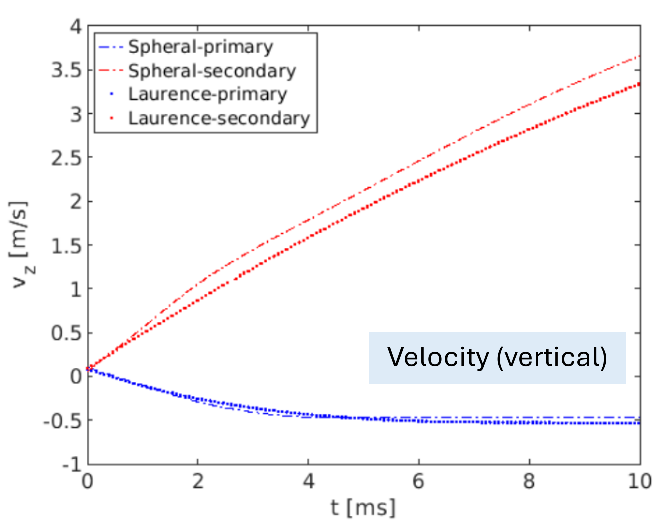
Simulation

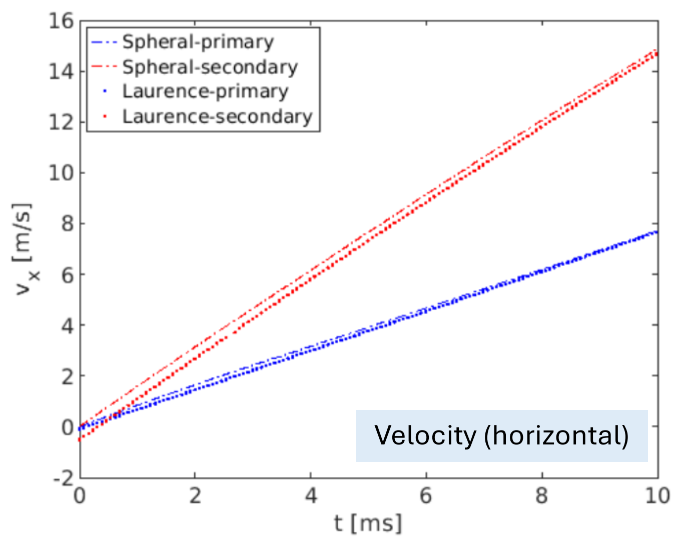
Experiment

[1] Laurence, Parziale, and Deiterding, "Dynamical Separation of Spherical Bodies in Supersonic Flow" (2012) J. Fluid Mech, 713, 159-182.

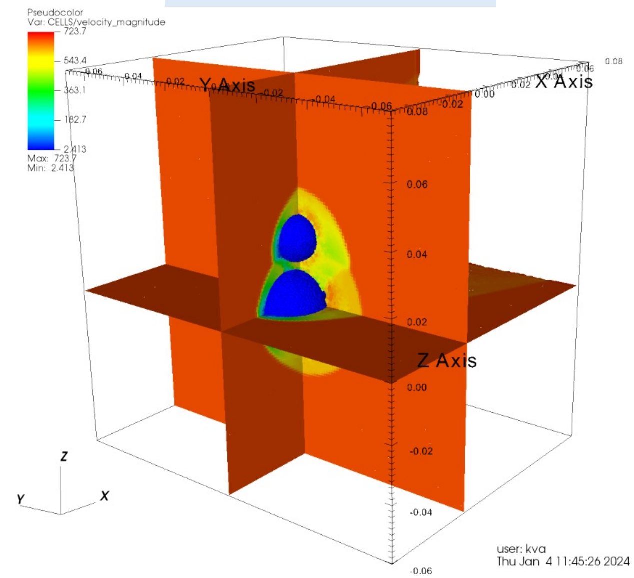
This experiment provides a nice test of capturing supersonic flow around solid bodies.

- Interacting solid bodies in high-speed flow, Laurence et al (2012)[1]
  - Align two Nylon spheres along the lateral axis
  - Flow air ($\gamma = 1.4$) at M ~ 4 along the longitudinal axis for approx. 10 [ms]
  - Features: Bow-shock 'riding' and shock-shock interaction
  - Analogous to the physics of an asteroid body breaking apart in atmosphere





Velocity magnitude

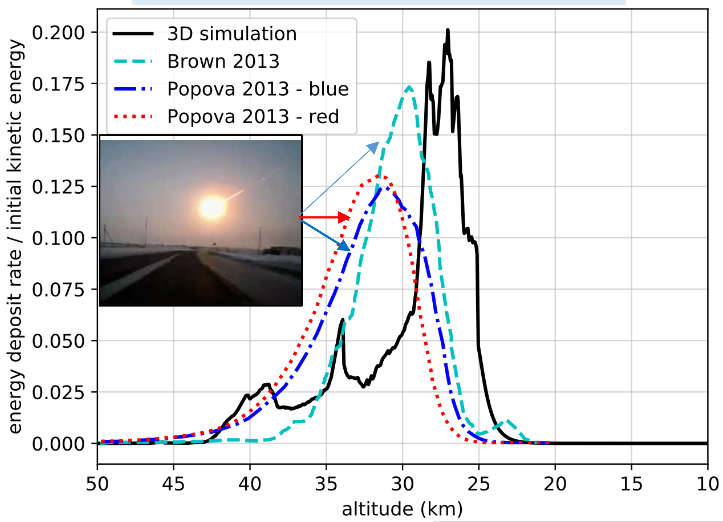


[1]Laurence, Parziale, and Deiterding, "Dynamical Separation of Spherical Bodies in Supersonic Flow" (2012) J. Fluid Mech, 713, 159-182.

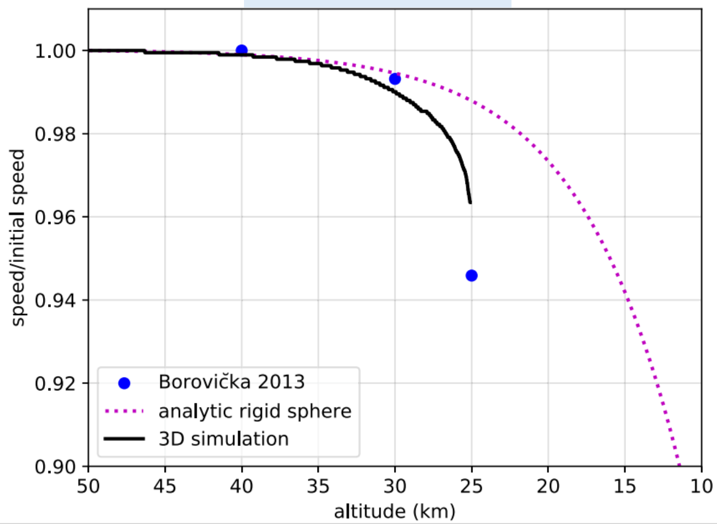
The airburst over Chelyabinsk in 2013 is a real-world example of the events we're working to understand.

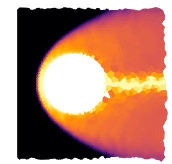
- FSISPH simulations of this event match observations for velocity and energy deposition.
  - Simulated peak luminosity 26-27km, slightly lower than observed peak luminosity of 29-32km based on reconstructions of Brown[2] and Popova[1].
  - Leading fragment speeds match observations of Borovicka[3] 2013

Atmospheric energy deposition rate

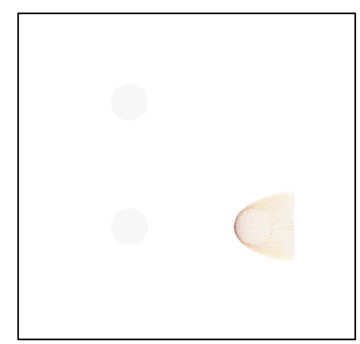


Bolide velocity





2D model



3D model

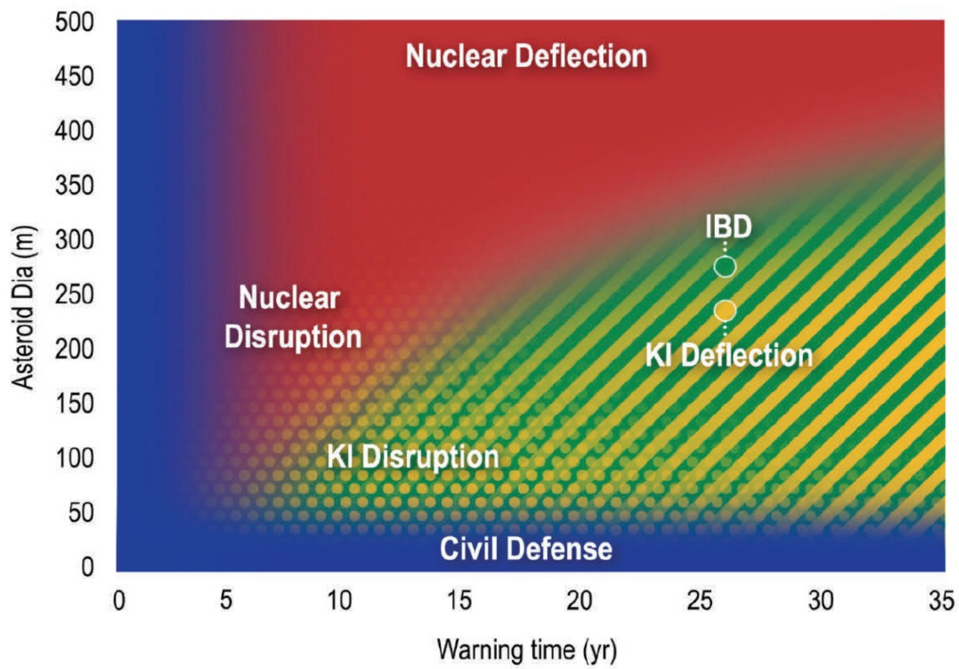
[1]**O. Popova**, et al., Chelyabinsk airburst, damage assessment, meteorite recovery and characterization, Science 342 (2013).
[2]**Brown, P.G** et al., A 500-kiloton airburst over Chelyabinsk and an enhanced hazard from small impactors, Nature 503 (7475), 238–241, (2013).
[3]**Borovička, J.**, et al., The trajectory, structure and origin of the Chelyabinsk asteroidal impactor, Nature 503, 235–237, (2013).
[4]**Pearl, J.M., et al.**, 2023. Insights into the failure mode of the Chelyabinsk meteor from high-fidelity simulation. Icarus 404.

Planetary Defense: diverting hazardous asteroids

Work with Kathryn Kumamoto, Jason Pearl, Veronika Korneyeva, Megan Bruck Syal, Mary Burkey, Rob Managan, and others

We are researching methods to mitigate (or better understand) the risks from asteroids impacting the Earth.

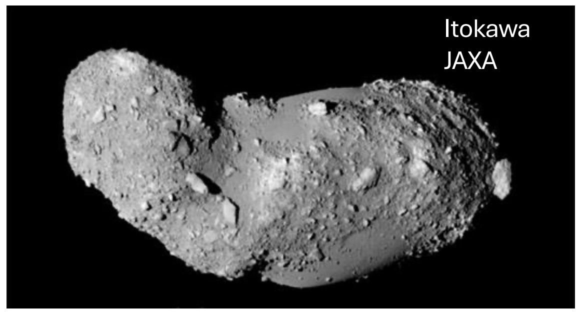
- FSISPH was developed to study the risk of airburst from bolides as part of this overall research.
- The best response depends on the warning time and size of the object.
  - Small object: civil defense preparation
  - For progressively larger objects (or shorter warning times) we have to escalate our response:
    - Ion-beam/gravity tractor ("slow push")
    - Kinetic impactor (KI) deflection
    - Nuclear Explosive Device (NED) deflection
    - KI or NED disruption for short warning times

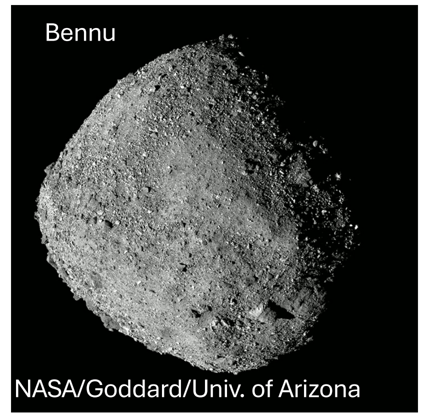


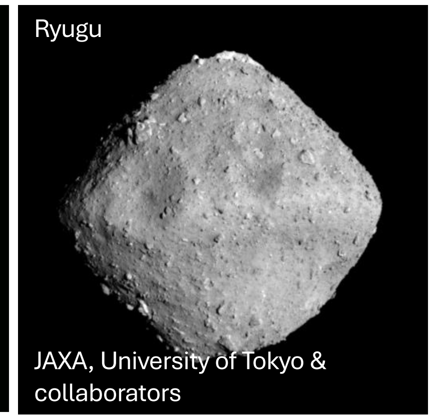
Credit: Decadal Strategy for Planetary Science and Astrobiology 2023-2032

Asteroids are not simple objects: most in the hazardous size range are most likely “rubble-piles”.

- Recent missions such as DART (Dimorphos), Osiris-REx (Bennu), and Hayabusa I and II (Itokawa and Ryugu) have found rubble-pile structures to be the norm.







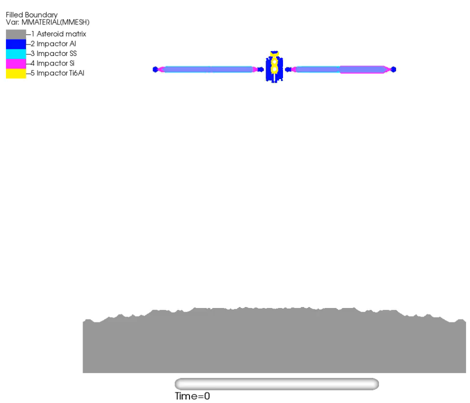
- Much of the risk is due to objects ~100’s of m in diameter.
  - Self-gravitation is very low
  - Escape velocities are typically ~10’s cm/sec
  - Loosely bound, highly porous (ϕ ~ 20%-80%) objects
    - Porosity probably a mixture of micro- and macro-porosity: i.e., porous boulders with voids between them.
- Rule of thumb: imparting Δv > 10% escape velocity risks disruption of the body!

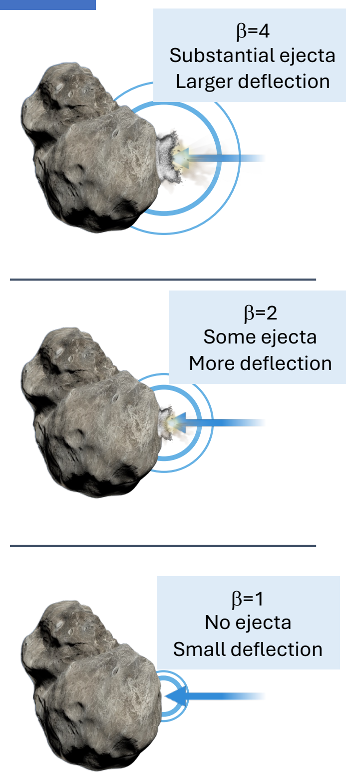
Kinetic impactors or nuclear explosives are the best understood options for deflecting an asteroid.

Kinetic Impactor

- Kinetic impactor momentum transfer enhanced by ejecta

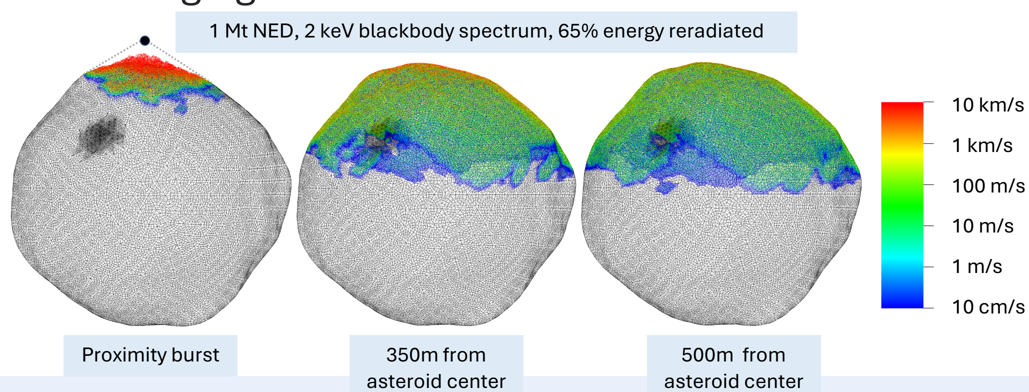
$$\beta \equiv 1 + \frac{m_{\text{ejecta}} v_{\text{ejecta}}}{m_{\text{spacecraft}} v_{\text{spacecraft}}}$$





Nuclear Explosives

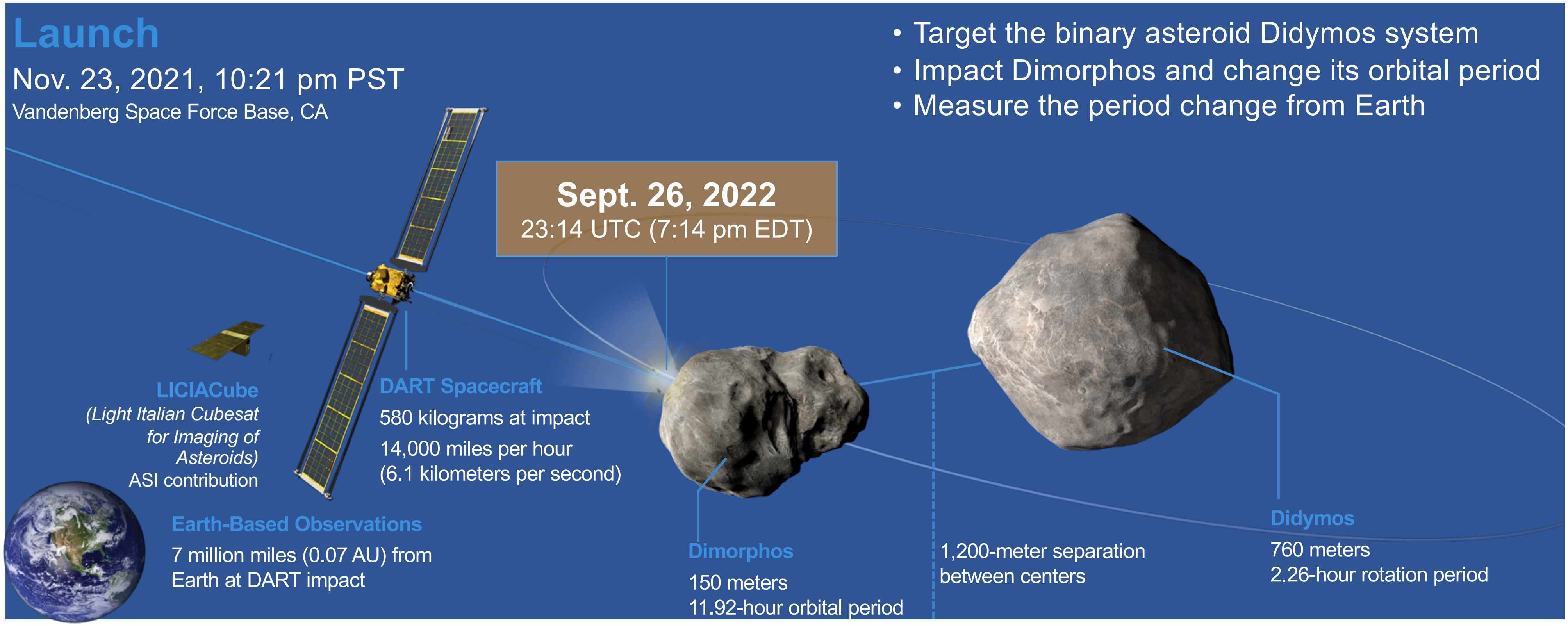
- Nuclear deflection uses energy deposited by a standoff explosion (via X-rays) to heat and ablate material from asteroid surface.
- Ablated material (and material ejected by the subsequent shock) imparts momentum to the asteroid.
- The effective push applied is adjustable by changing standoff distance.



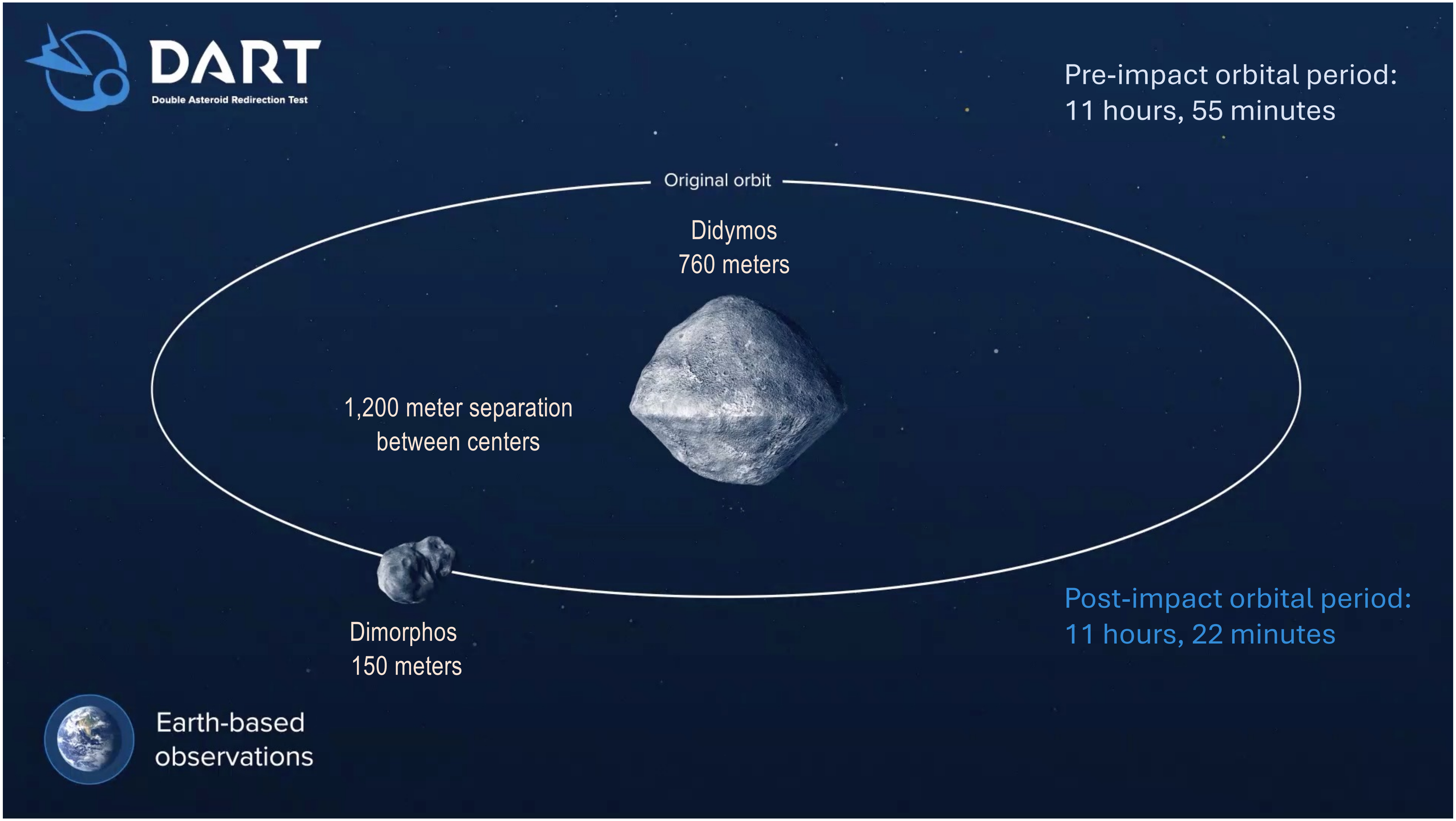
Damage, fracture, and ejecta modeling are key challenges in understanding asteroid mitigation.

- This requirement is a big reason meshfree methods are useful for this problem – robust Lagrangian discretizations are very well-suited for following damage, fracture, and ejecta.
- We also have extreme resolution requirements:
  - Overall objects are 100's of meters to a few kilometers in size
  - KI impact modeling typically requires on the order of cm's resolution
  - Resolving ablation from X-rays is even worse: we require microns at the ablating surface
- For impact problems we typically feather the resolution from fine at the impact site to coarser with distance.
  - Requires generalizing standard damage modeling algorithms (like Benz-Asphaug model) to properly allow varying resolution.
- The nuclear ablation surface additionally requires ASPH (Adaptive SPH with elliptical smoothing volumes) with fine resolution aligned with depth at the surface.
  - Aspect ratios for some cases are ~10^4-10^5, which required new ASPH development as well.

NASA's DART mission was humanity's first planetary defense test mission and demonstrated a kinetic impact



Slide credit: DART team


DART
Double Asteroid Redirection Test
Pre-impact orbital period:
11 hours, 55 minutes
Original orbit
Didymos
760 meters
1,200 meter separation
between centers
Dimorphos
150 meters
Post-impact orbital period:
11 hours, 22 minutes
Earth-based
observations

DART camera feed on final approach to Dimorphos

Reproducing all aspects of the DART impact requires modeling the complex geometry and materials of the system.

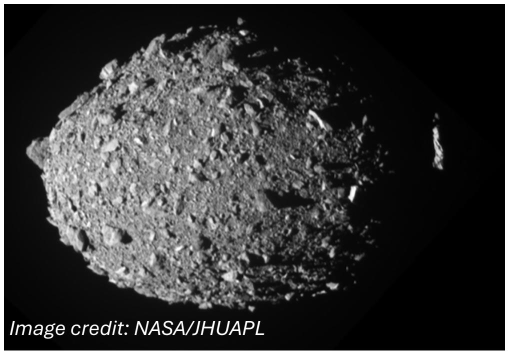
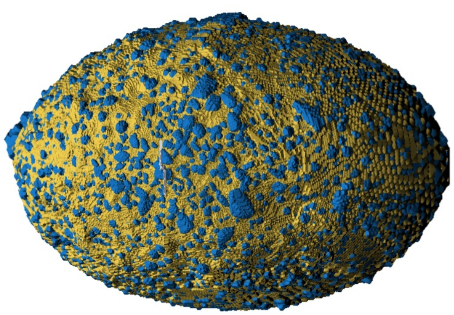
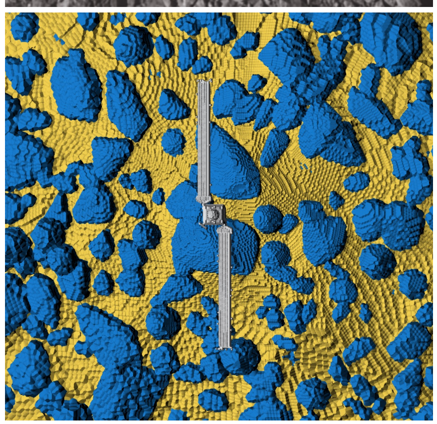


Image credit: NASA/JHUAPL

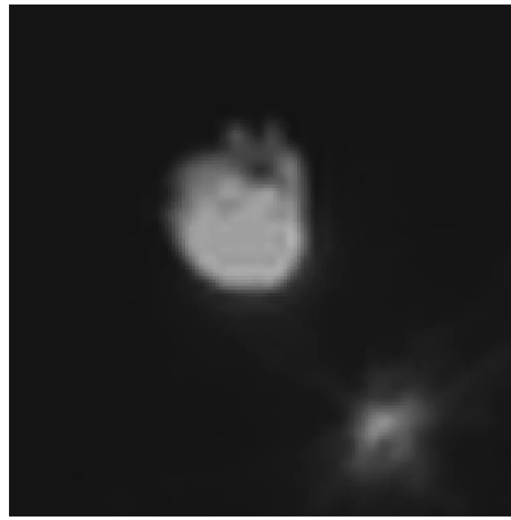


Yellow = matrix

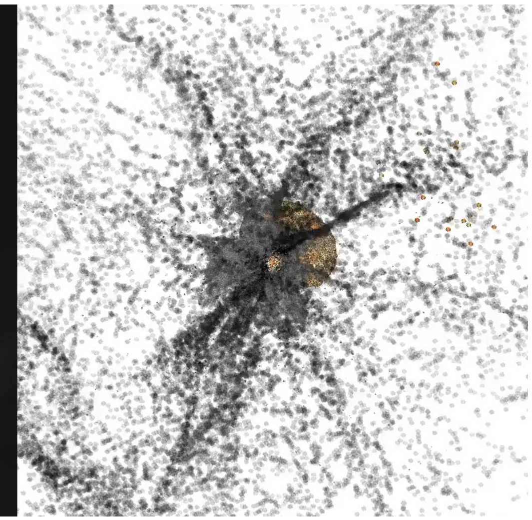
Blue = boulders



LICIACube images



Spheral simulation



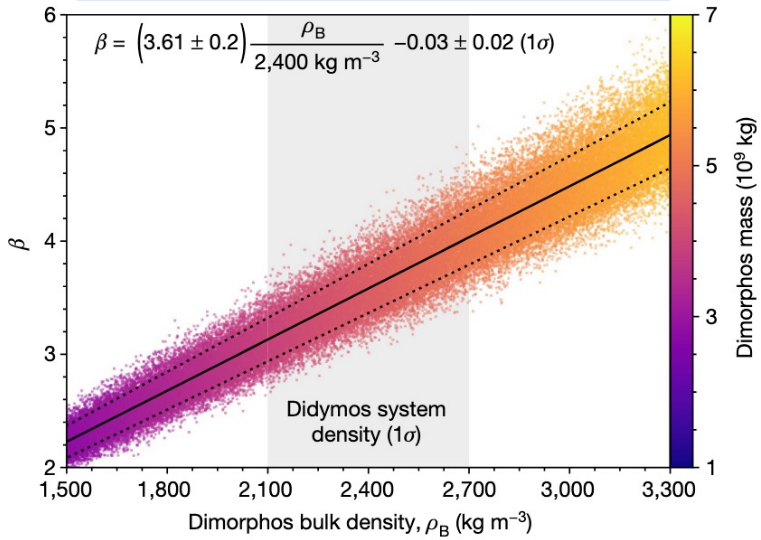
Simulations to match the outcome require:

- Multiple materials: Matrix, boulders, spacecraft (Al, SS, Ti, Si)
- Precisely recreating local topography and spacecraft geometry
- High resolution at the impact site
- Analyzing a degenerate, multi-dimensional space

Modeling is essential to understanding the full outcome of the DART mission, but is challenging.

- Much comes down to material modeling (cohesion, porosity, yield strength intact and damaged, damage parameters, etc.)
  - These are all unknown to various degrees for asteroids and so involve multi-dimensional parameter sweeps.
  - Incredibly weak objects with low-gravity, so ejecta production goes on for many minutes
    - Simulating to such late times with explicit times-steps in the μsecs adds to the challenge.
- Our primary concern for planetary defense (PD) is the momentum enhancement (β).
  - Unfortunately this measurement is degenerate until we get a measurement of Dimorphos' density.
  - Δv ~ 2.7 mm/sec from orbital period change
  - ESA Hera mission should tell us more!

β as a function of Dimorphos' density[1]

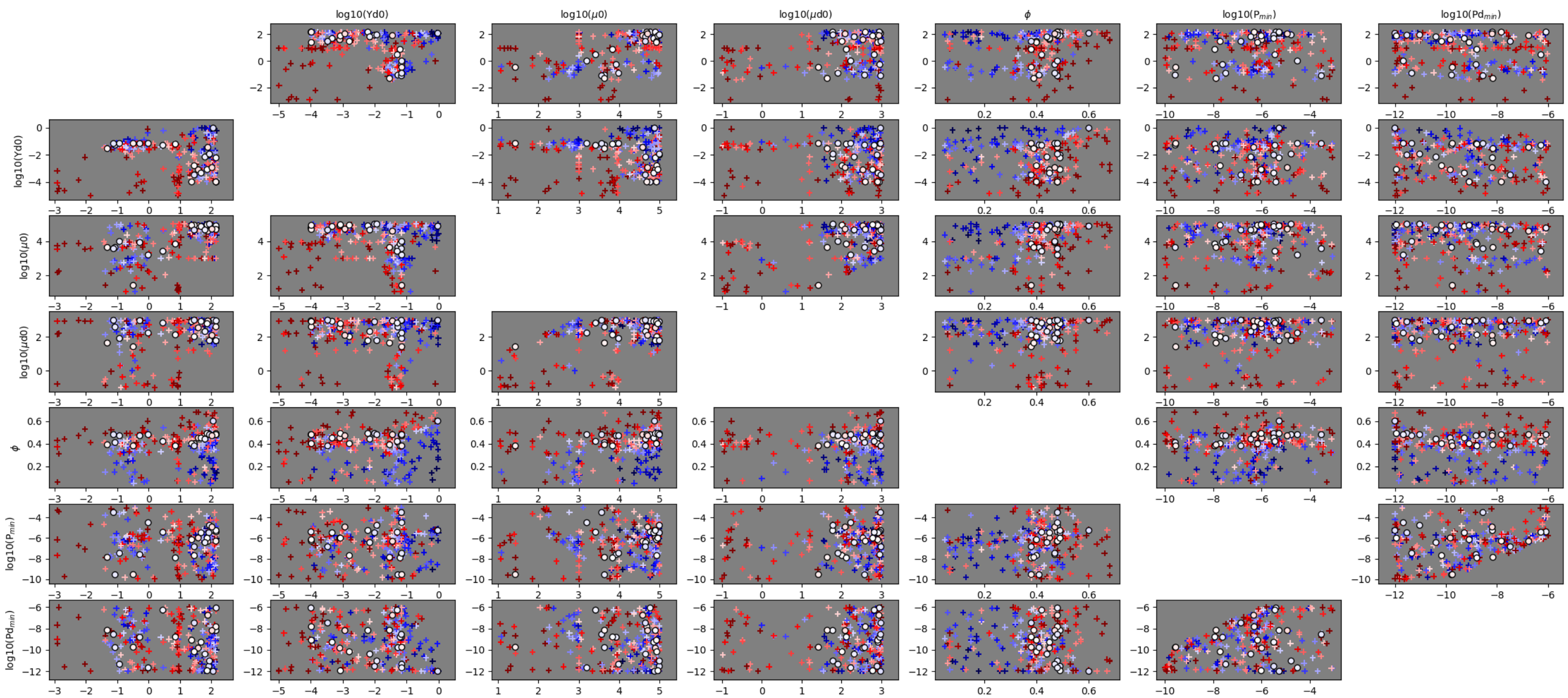


[1]Cheng, A.F.,et al., 2023. Momentum transfer from the DART mission kinetic impact on asteroid Dimorphos. Nature 616, 457–460. https://doi.org/10.1038/s41586-023-05878-z

Testing the phase space of possible material parameters requires many hundreds of 3D models.

- The robustness of meshfree models is key – no babysitting calculations that need to be launched automatically.

Parameter sweep of material properties to match observed deflection[1]

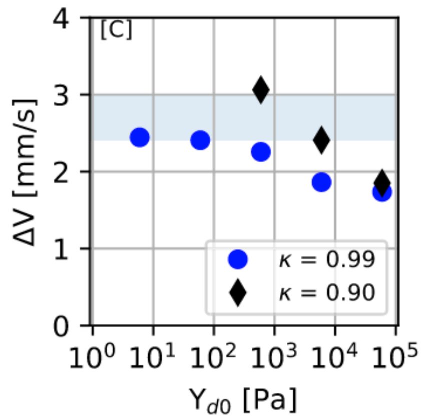


[1]Kumamoto, K.M., Owen, J.M., Syal, M.B., Pearl, J., Raskin, C., Caldwell, W.K., Rainey, E., Stickle, A.M., Daly, R.T., Barnouin, O., 2022. Predicting asteroid material properties from a DART-like kinetic impact. Planet. Sci. J. 3. https://doi.org/10.3847/PSJ/ac93f2

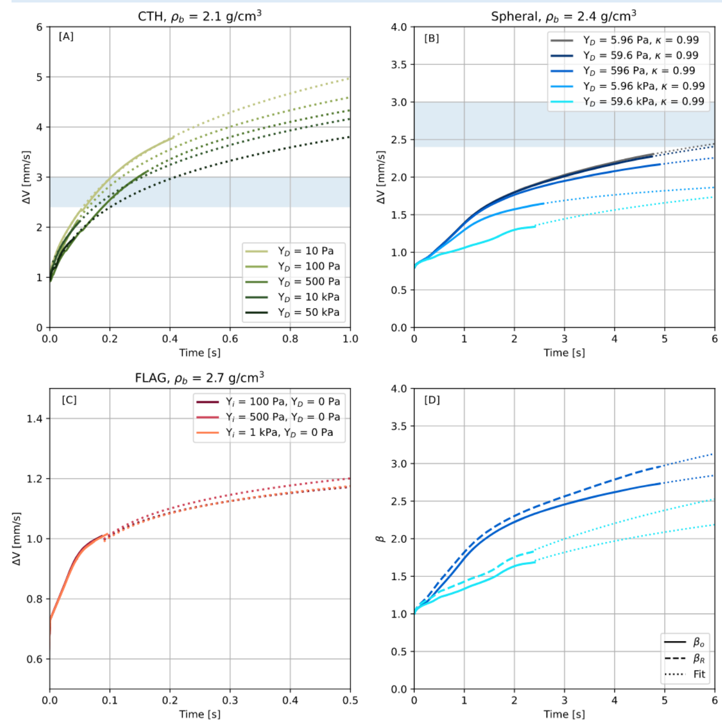
Varying material properties we can match the observed deflection velocity from DART.

- However, we must also match all properties we know of
  - The observed duration of ejecta production and overall total mass ejected
  - The final crater size and shape (TBD by Hera?)
- Our overall goal is to understand the implications for planetary defense.
- It appears $\beta \approx [2, 3]$, but this is only one data point!
  - Asteroid properties have large variations between different populations.
  - One point is better than none!

Extrapolated Δv for different porosity compaction laws[1]



Δv time histories for different material strengths[1]



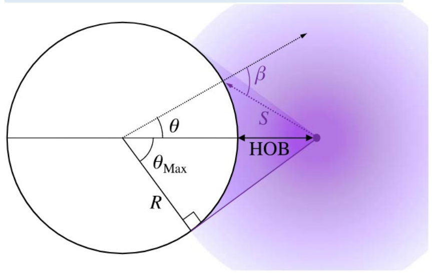
[1]Stickle, A.M. et al., 2025. Dimorphos's Material Properties and Estimates of Crater Size from the DART Impact. Planetary Science Journal 6. https://doi.org/10.3847/PSJ/ad944d

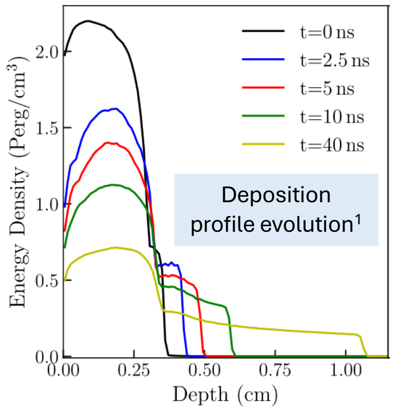
Nuclear deflection involves two phases dominated by different physics on very different timescales.

X-Ray Energy Deposition

- X rays penetrate 1 μm – 1 cm into the material, causing heating and ionization. Some energy re-radiates away.
- Only a full radiation-hydrodynamics code can cover all the physics that is happening in this process.
- Timescales in the nanosec – μsec range

Geometry for deposition surface[1]



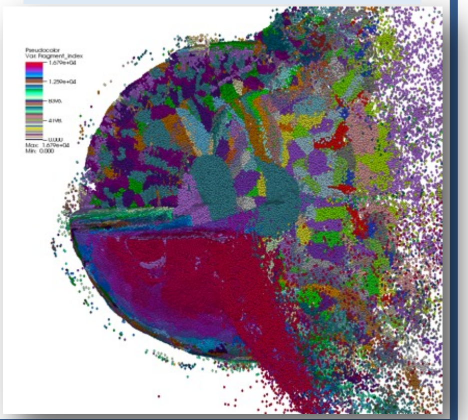


Deposition profile evolution[1]

Hydrodynamics

- Everything that happens after the energy deposition.
- The deposited energy causes material to begin moving and expanding
- At this point, a standard hydrocode can follow the material's movement and energy.
- Shock physics, fracture, failure, and ejecta dominate evolution.
- Timescales from seconds -- hours

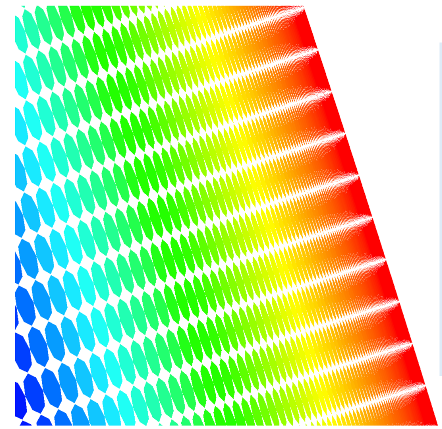
Robust disruption and dispersal of fragments due to shock from ablation physics



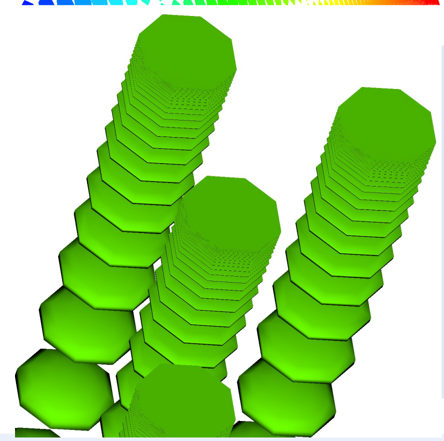
[1]Burkey, M.T., Managan, R.A., Gentile, N.A., Syal, M.B., Howley, K.M., Wasem, J.V., 2023. X-ray energy deposition model for simulating asteroid response to a nuclear planetary defense mitigation mission. Planet. Sci. J. 4, 243. https://doi.org/10.3847/PSJ/ad0838

The major challenge with following the ablation driven shock and subsequent evolution is the required resolution.

- In order to resolve the energy deposition profile with depth we need resolutions ~μm.
- The overall asteroid shapes are typically 100's m across, so uniform resolution is unobtainable.
- Fortunately the ablation driven shock rapidly relaxes these resolution constraints away from the surface.
- Adaptive SPH[1,2] with arbitrary ellipsoidal kernel shapes makes this possible.
- Even with ASPH is this an extremely challenging problem, and we are still developing improvements to handle this problem.
  - Compatible SPH, FSISPH, and CRKSPH are all effective for this problem.



2D ASPH elliptical kernels



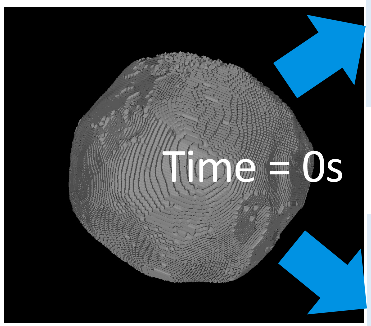
3D ASPH ellipsoidal kernels

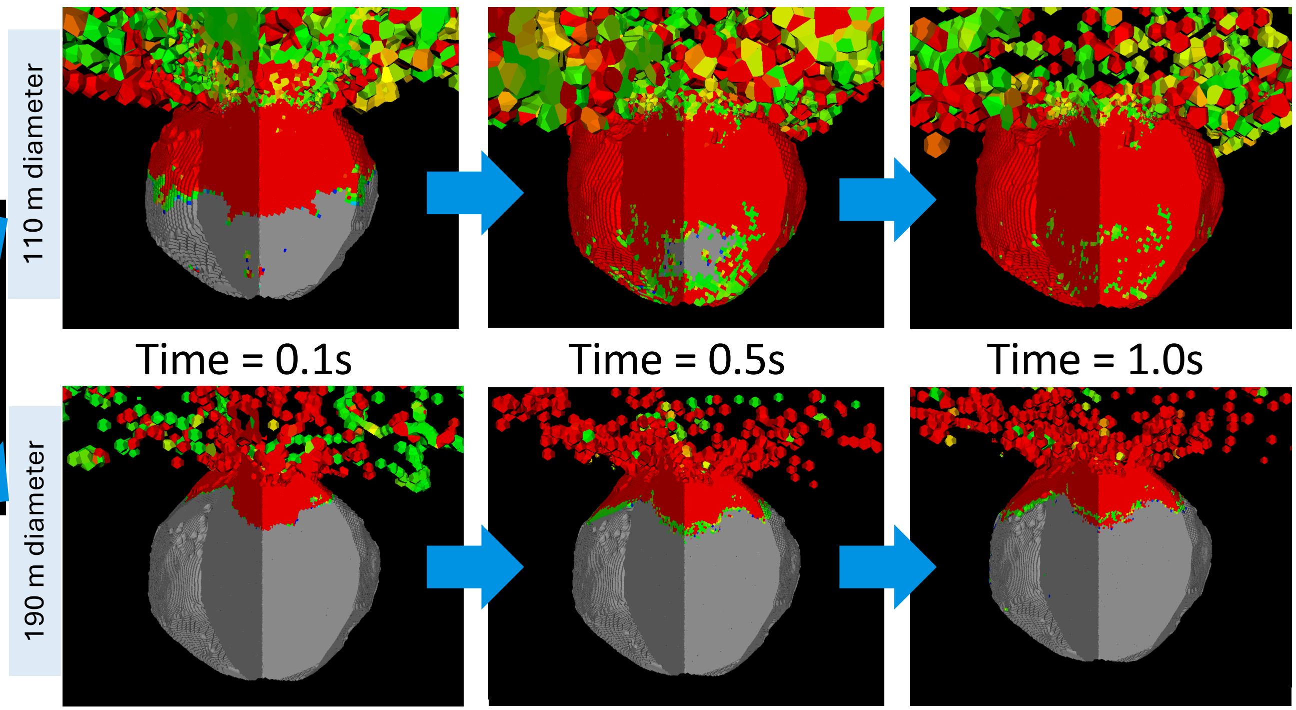
[1]Owen, J.M., Villumsen, J.V., Shapiro, P.R., Martel, H., 1998. Adaptive smoothed particle hydrodynamics: methodology. II. Astrophys. J. Suppl. Ser. 116, 155–209.
[2]Owen, J.M., 2010. ASPH modeling of material damage and failure, in: Proceedings of the 5 Th International SPHERIC Workshop. Manchester, UK, pp. 297–304.

Whether we deflect or disrupt an asteroid depends on both standoff distance and asteroid size.

- We may not know all the properties of the asteroid before encounter!

1 Mt yield
10 m standoff
SiO_2 target





= fully damaged

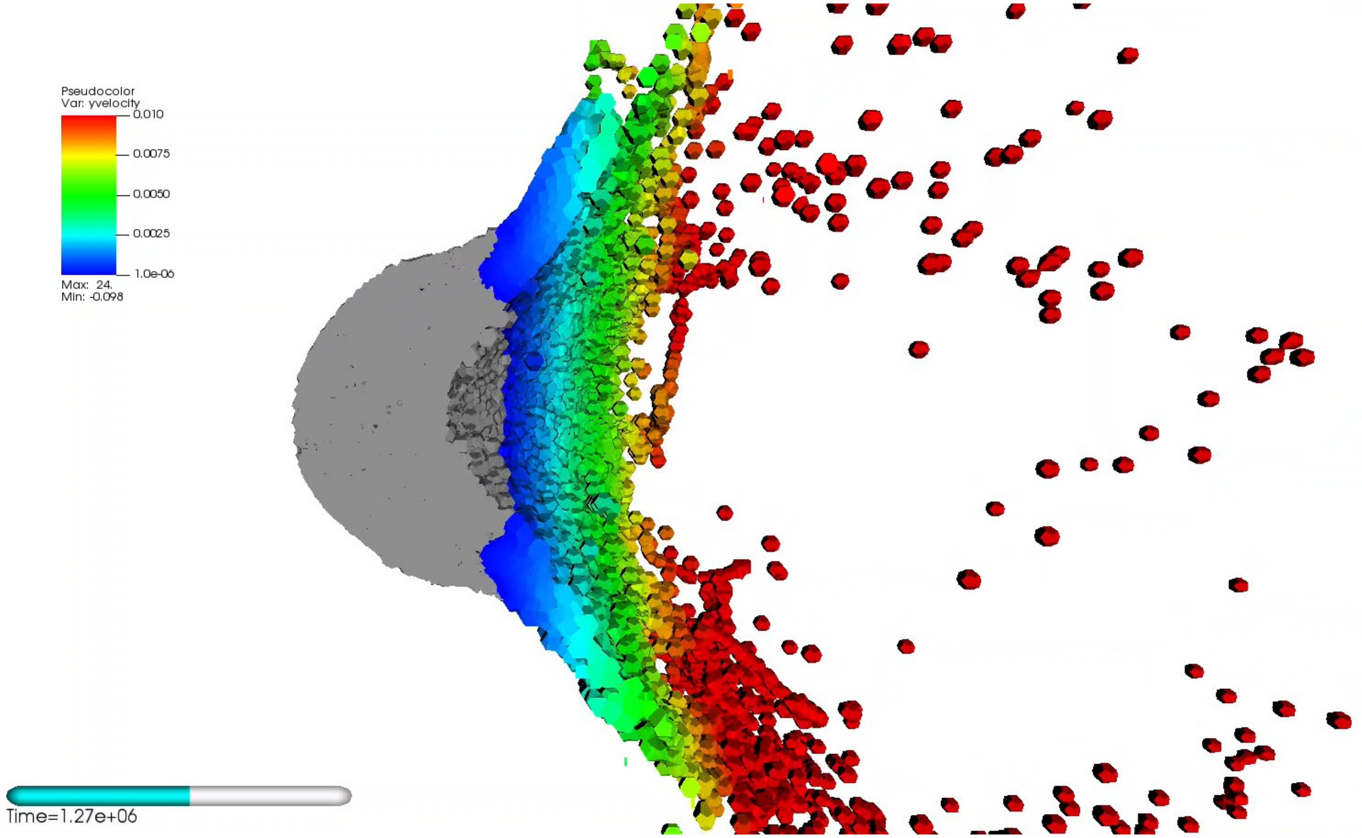
Fully disrupted at 1s

<1% of material reaches Earth if disrupted 1 month from impact

Remnant velocity is 1.5 m/s at 2s.
(V_{esc} = 0.1 m/s)

90% of material reaches Earth if disrupted 1 month from impact

When in doubt or the timeline is short, disrupting the target asteroid robustly may be the best option.



Model performed by Mary Burkey

Designing the best discretization is not a one solution fits all proposition.

- Broadly speaking meshfree methods can indeed be applied to many sorts of problems.
  - For the cases I've focused on having a robust Lagrangian reference frame is a key advantage.
- Writing the discrete form of the PDE's involved to reproduce the most important properties of the continuum equations is often central to successful models.
  - The long history of mimetic methods is well worth studying for such principles.
- Tradeoffs are necessary – we generally can't have everything.
  - Consistency/accuracy vs. conservation (RK vs. CRKSPH)
  - Robustness vs. higher-order accuracy (SPH variations vs. Reproducing Kernel variants)
  - Robustness to wild differences in materials and densities (FSISPH and the choices therein)
- It seems obvious but is worth repeating, know your problem!
  - What are the important aspects to maintain in your discretization?

Flexibility in your code base is useful to support such varied applications.

- Our code (Spheral[1]) is designed to make developing, maintaining, and applying a variety of methods such as we've talked about today natural.
  - Hybrid C++/Python environment – methods can be implemented in either just as easily.
  - Python scriptable interface allows the choice of just the methods you want for the problem at hand.
    - Each of the methods discussed today are available in Spheral (with the exception of the radiation diffusion).
    - The user simply constructs the packages they want to use which can be added to an arbitrary choice of time advancement algorithms.
- New methods are being developed and tested all the time.
  - Discrete Element Method recently added.
  - Riemann SPH (GSPH as well as particular variants like MFM[2] and MFV[2])
  - SVPH (a hybrid which uses a Voronoi tessellation to construct the differencing method)

[1] https://github.com/LLNL/Spheral

[2] Hopkins, P.F., 2015. A new class of accurate, mesh-free hydrodynamic simulation methods. Monthly Notices of the Royal Astronomical Society 450, 53–110.