Introduction to SPH

A selective tour of things to be aware of as you enter the SPH world!

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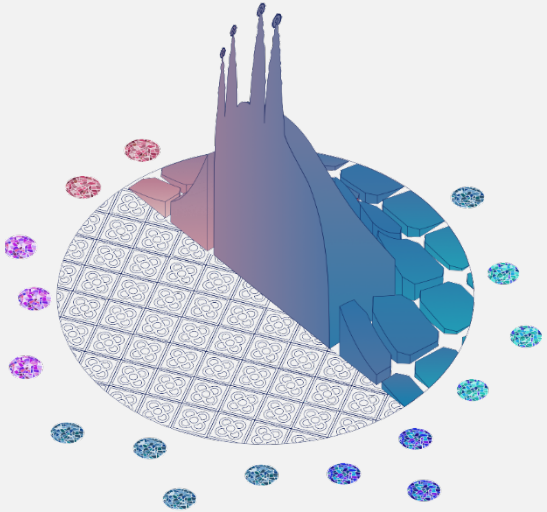
SPHERIC

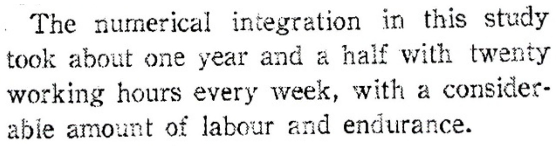


1. Consistency, stability, conservation, convergence
2. (In)compressibility
3. Walls
4. Some SPH variants

Timescales (very approximate)…

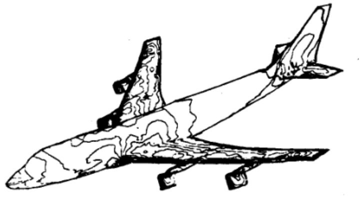
FVM,FEM

Kawaguti



The numerical integration in this study took about one year and a half with twenty working hours every week, with a considerable amount of labour and endurance.

Jameson



CFD by hand
1954

Panel methods
1960s

SIMPLE incompressible
1972

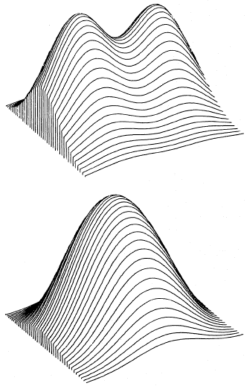
aircraft Euler
1982

commercial FVM
~1988

FOSS FVM
~2004

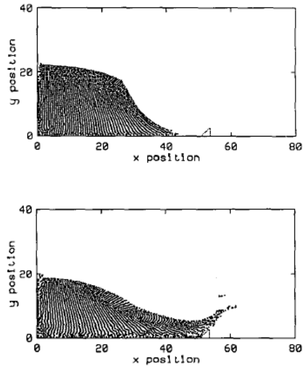
2025

1977
birth of SPH



Monaghan & Gingold

1994
the dam breaks!



2005

SPHERIC

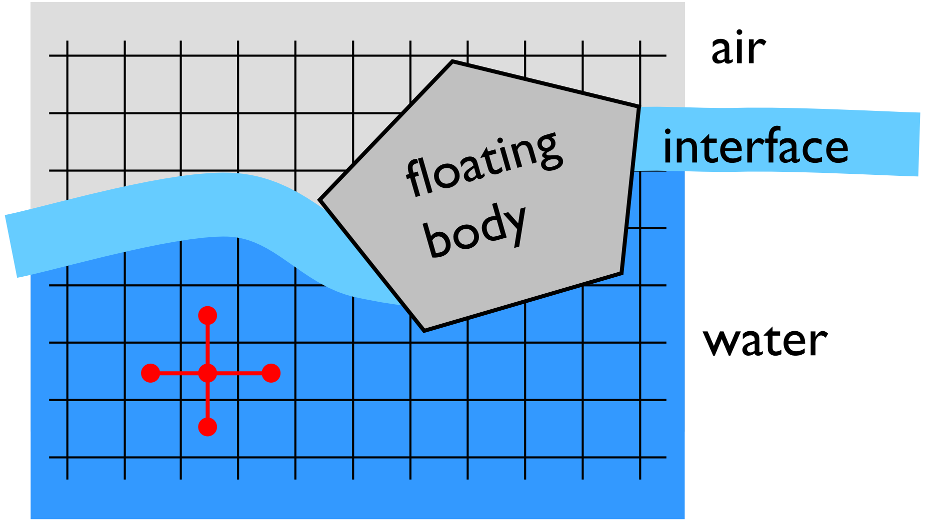
FOSS commercial

SPH

SPH is now older than FVM was when fully embedded in industry!

The particle alternative

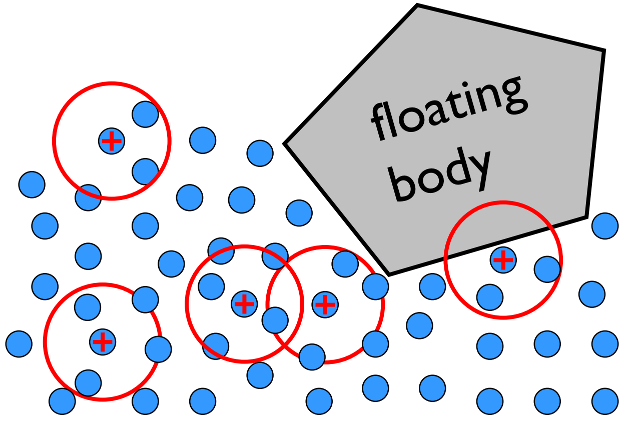
Mesh-based



air

water

Particle



air need not
be modelled

no explicit
model of
interface

water
“particles”

Solving the Euler* equations

Eulerian

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{\rho} \nabla p = 0$$

Lagrangian

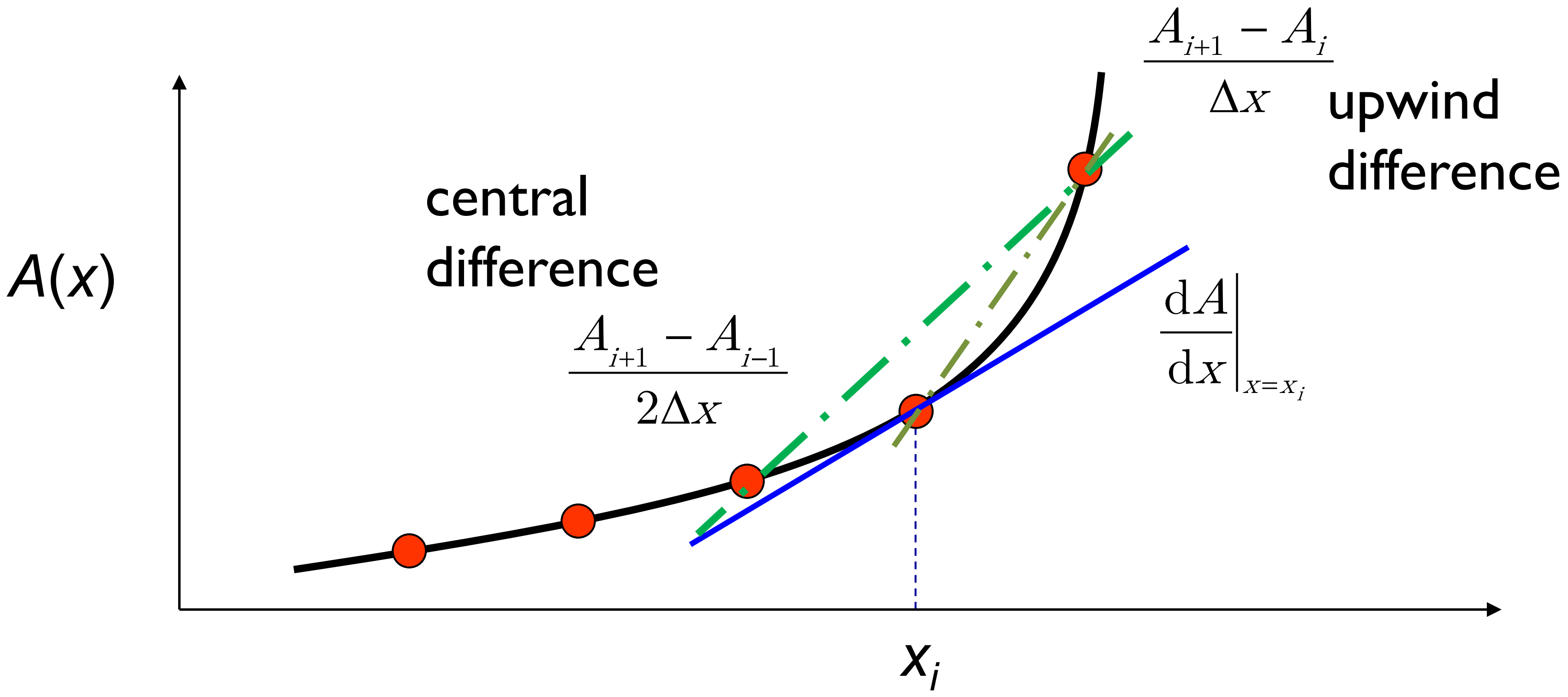
$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho \nabla \cdot \mathbf{u} = 0$$

$$\frac{\mathbf{du}}{\mathbf{d}\boldsymbol{t}} + \frac{1}{\rho} \nabla p = 0$$

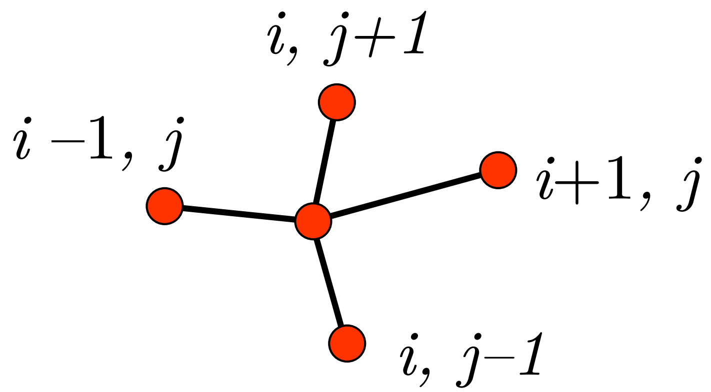
We have to estimate spatial gradients using function data at discrete points.

*I won't deal with viscous stresses in this lecture.

Estimating gradients on a mesh



2D



Stencil:
Topological set of points used for calculations

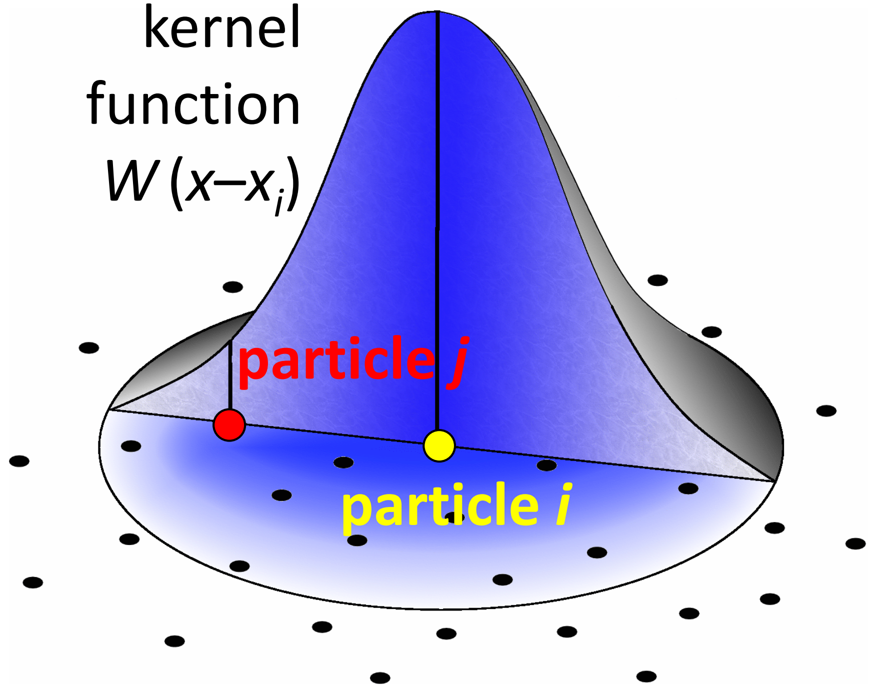
Estimating gradients without a mesh

Smoothing estimate of $A(\mathbf{x})$ at $\mathbf{x}$:

$$\left\langle A\left(\mathbf{x}\right)\right\rangle = \sum_j A_j W\left(\mathbf{x}-\mathbf{x}_j\right) V_j$$

Gradient estimate:

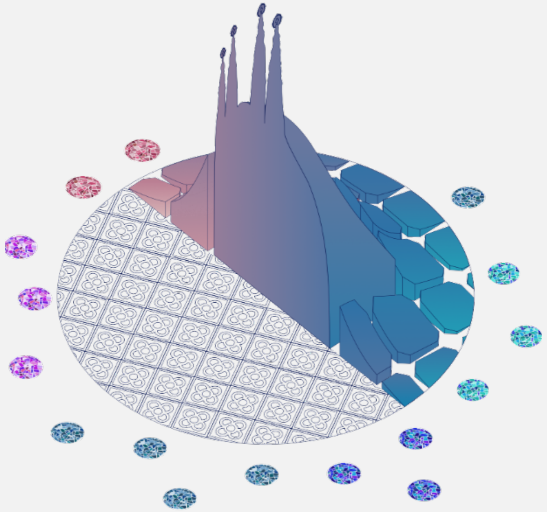
$$\left\langle \nabla A\left(\mathbf{x}\right)\right\rangle = \sum_j A_j \nabla W\left(\mathbf{x}-\mathbf{x}_j\right) V_j$$



Typical kernel features:

- Compact support
- radius 2h
- Smooth
- Radially symmetric
- Normalised s.t. $\int W \mathrm{d}\boldsymbol{x} = 1$

1. **Consistency, stability, conservation, convergence**
2. (In)compressibility
3. Walls
4. Some SPH variants



What is an accurate solution?

Lax Equivalence Theorem:

consistency + stability $\Rightarrow$ **convergence**

consistency: discrete scheme $\longrightarrow$ PDE as $\Delta x,\ \Delta t \longrightarrow 0$

stability: small perturbations decay

convergence: discrete solution $\longrightarrow$ *something* as $\Delta x,\ \Delta t \longrightarrow 0$

Lax-Wendroff Theorem:

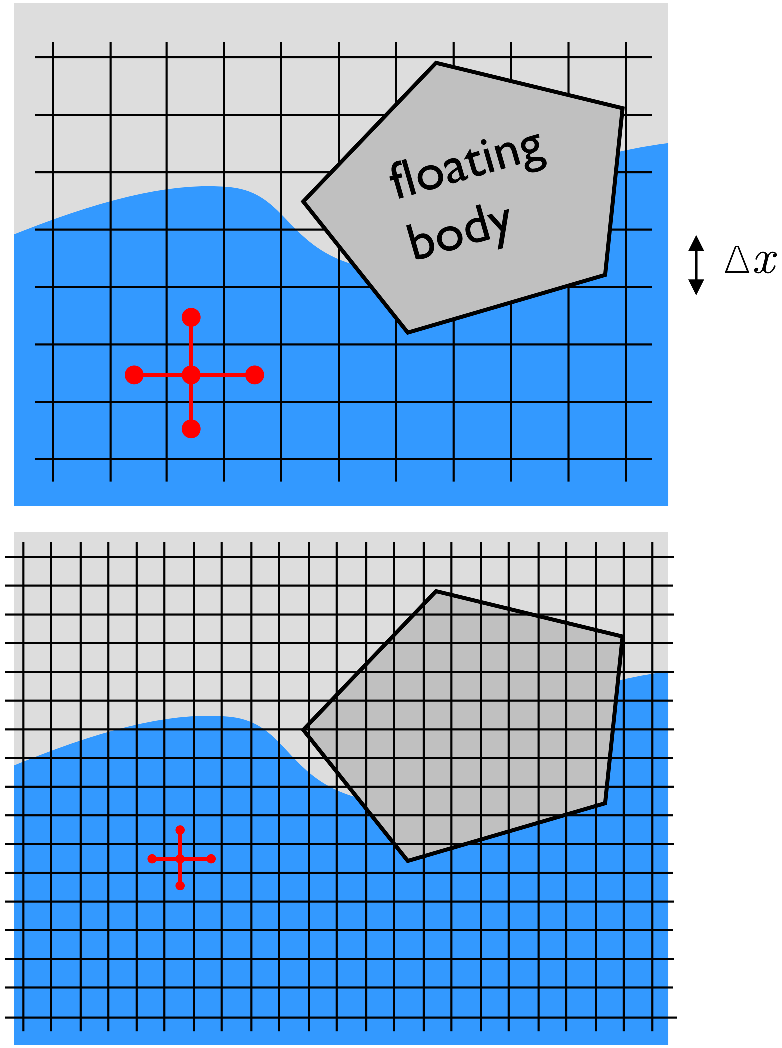
convergence + conservation $\Rightarrow$ ***weak* solution to PDEs**

conservation: exact

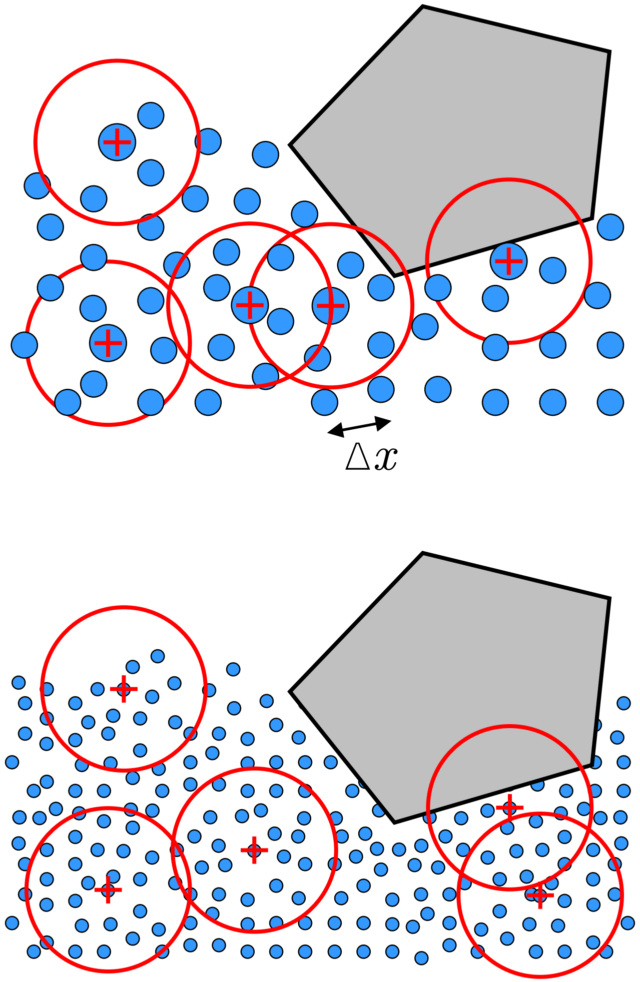
weak solution: integral over space

What does as $\Delta x \rightarrow 0$ mean in SPH?

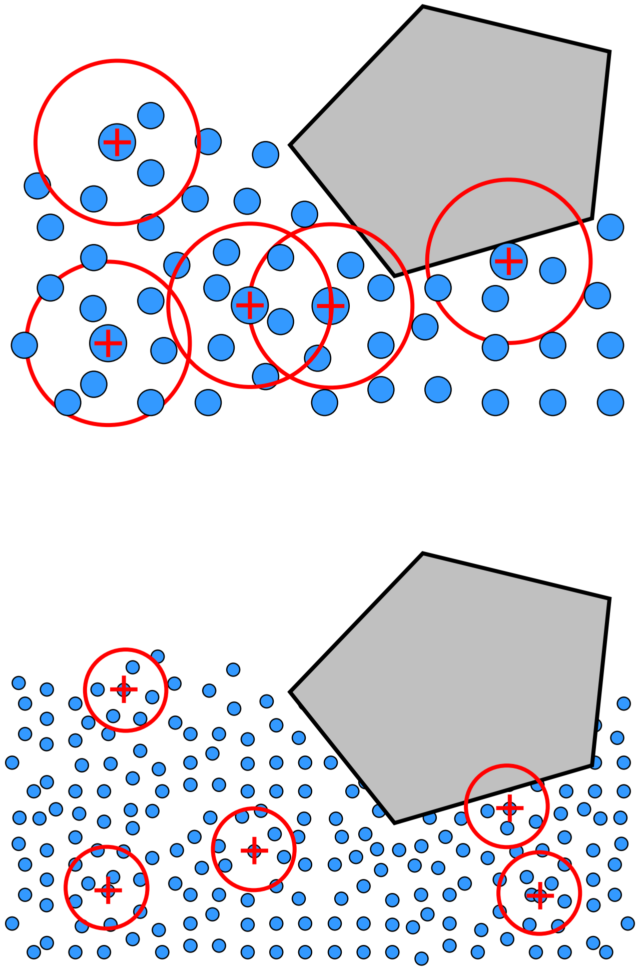
Mesh: reduce Δx
constant stencil



SPH: reduce Δx
constant h
changing $\Delta x/h$, n , "stencil"



SPH: reduce h
constant $\Delta x/h$, n , "stencil"



Two stages of approximation

$$\frac{\partial A(x_a)}{\partial x} \quad \underbrace{\cong -\int_{x_a-2h}^{x_a+2h} A(x)\frac{\partial W(x-x_a)}{\partial x}dx}_{\substack{\textbf{smoothing} \\ \textbf{(mollification)}}} \quad \underbrace{\cong -\sum_b A(x_b)\frac{\partial W(x_b-x_a)}{\partial x_b}\Delta x_b}_{\substack{\textbf{discretisation} \\ \textbf{(quadrature)}}}$$

Taylor series analysis – uniform particle spacing

SPH approximation

exact

$$-\sum_b A_b W_b' \Delta x_b = \left.\frac{\partial A}{\partial x}\right|_{x=x_a}$$

smoothing error

$$+\frac{h^2}{6}A_a'''\int s^3 \hat{W}' ds + \ldots$$

$$-\left(\frac{\Delta x}{h}\right)^{\beta+2}\frac{B_{\beta+2}}{(\beta+2)!}\left(1-2^{-\beta-1}\right)\left[A_a{}'\left(4\hat{W}^{(\beta+2)}_{s=2}+2(\beta+1)\hat{W}^{(\beta+1)}_{s=2}\right)+O\left(h^2\right)\right]-\ldots$$

discretisation error

Quinlan *et al.* (2006)

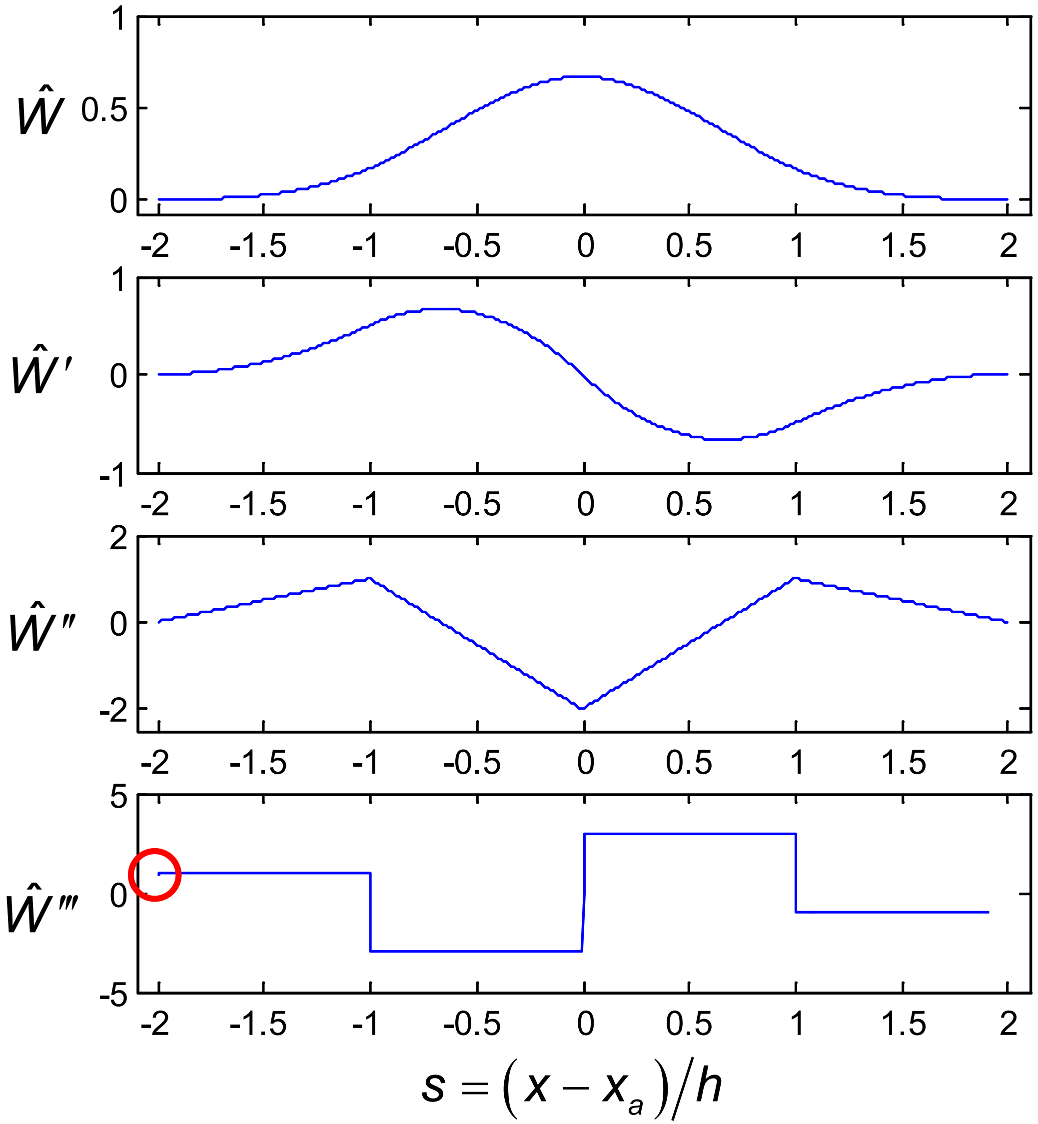
Kernel boundary smoothness β

Boundary smoothness of the kernel is the highest integer **β** for which the βth derivative and all lower derivatives are zero at the boundary of the compact support.

cubic B-spline: β = 2

Gaussian: β → ∞ in infinite domain

cubic B-spline kernel

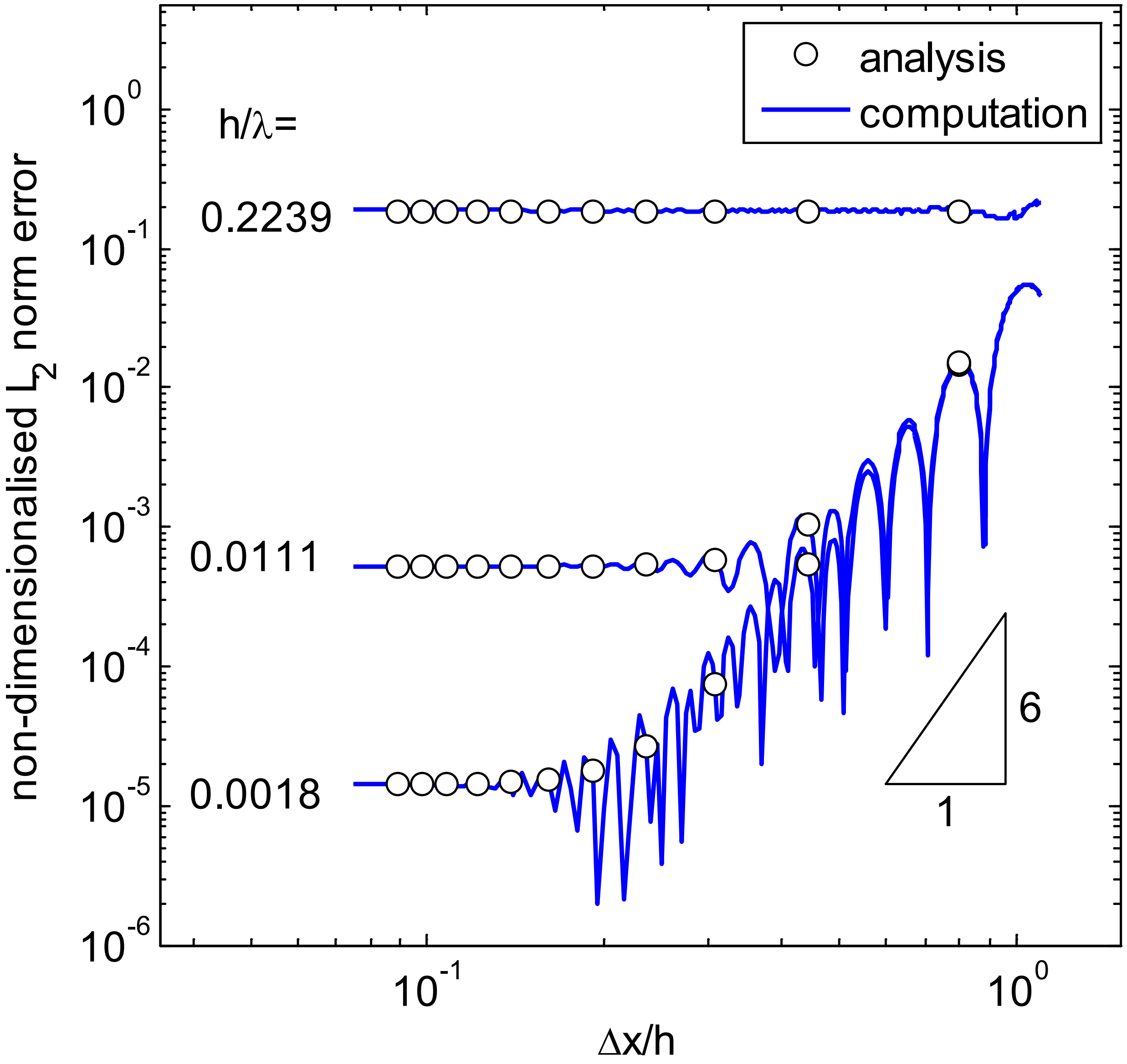


Consistency – constant h, changing $\Delta x/h$

sinusoidal test function *A*(*x*), wavelength λ

10th order polynomial kernel, β=4

When $\Delta x/h$ is small enough, no gains from further reduction.

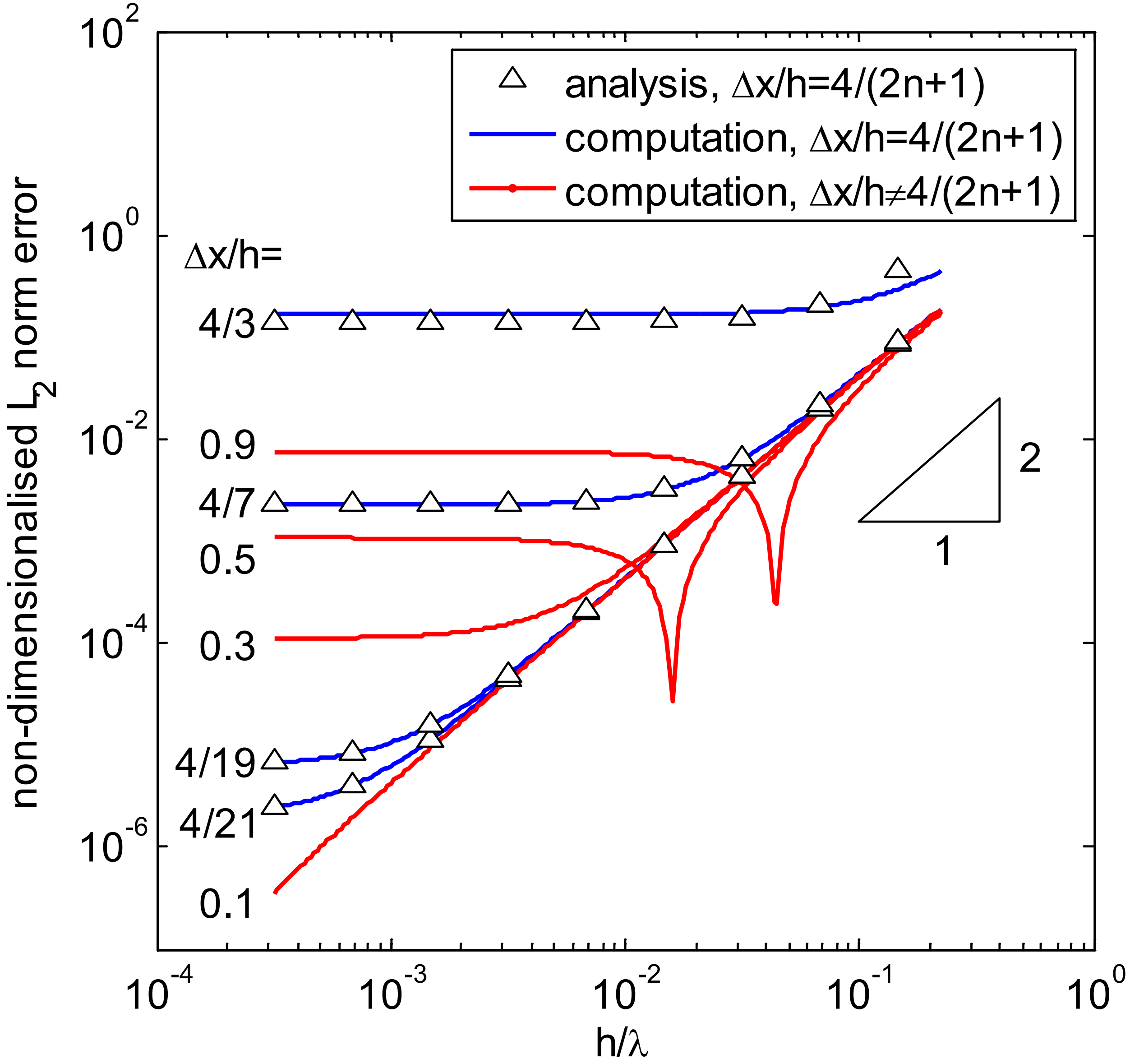


Consistency – changing h, constant $\Delta x/h$

sinusoidal test function *A*(*x*), wavelength λ

10th order polynomial kernel, β=4

Discretisation error sets a lower limit



Non-uniform particle spacing

$$
\begin{aligned}
-\sum_b A_b W'_b \Delta x_b = {} & \left.\frac{\partial A}{\partial x}\right|_{x=x_a} \\
& - A'_a\left(\int \hat{W} ds - 1\right) + \frac{h^2}{6} A'''_a \int s^3 \hat{W}' ds + \ldots \quad \Big\} \ \text{smoothing error} \\
& \left.\begin{aligned}
& -\frac{1}{h}\left[A_a \delta O\left(\left(\frac{\Delta x}{h}\right)^3\right) + \frac{A_a}{2}\left(\delta^2 + \frac{1}{12}\right) O\left(\left(\frac{\Delta x}{h}\right)^4\right)\right] \\
& -\left[A'_a \delta O\left(\left(\frac{\Delta x}{h}\right)^3\right)\right] \\
& -h\left[\frac{A''_a}{2}\delta O\left(\frac{\Delta x}{h}\right) + \frac{A''_a}{2} O\left(\left(\frac{\Delta x}{h}\right)^4\right)\right] - \ldots
\end{aligned}\right\} \ \text{discretisation error}
\end{aligned}
$$

where δ is a measure of deviation from uniform spacing

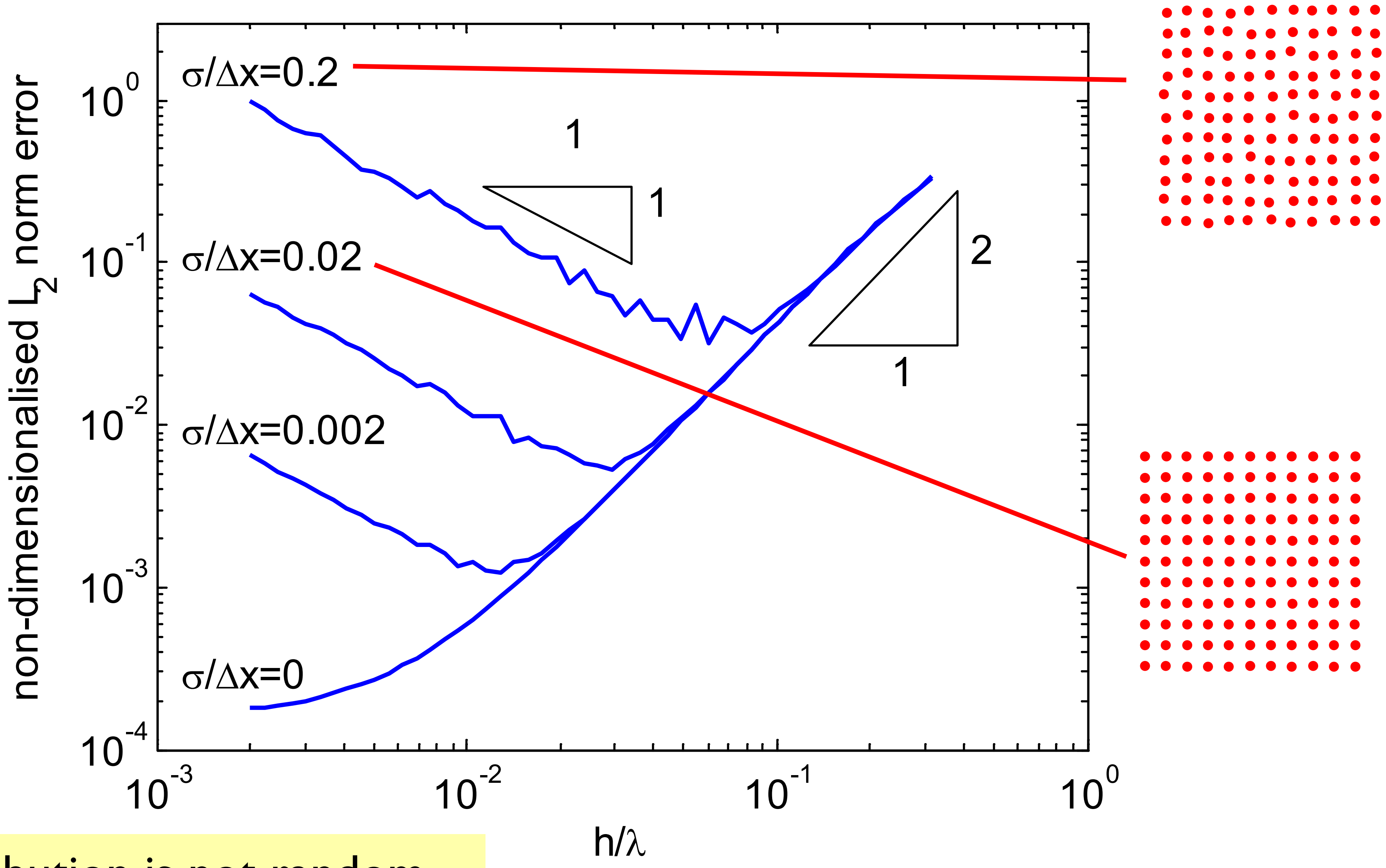
Non-uniform spacing – results

Normally distributed random perturbation of standard deviation σ imposed on uniform particle distribution.

sinusoidal test function, wavelength λ

10th order polynomial kernel, β=4

$\Delta x/h = 0.364$



But: SPH particle distribution is not random

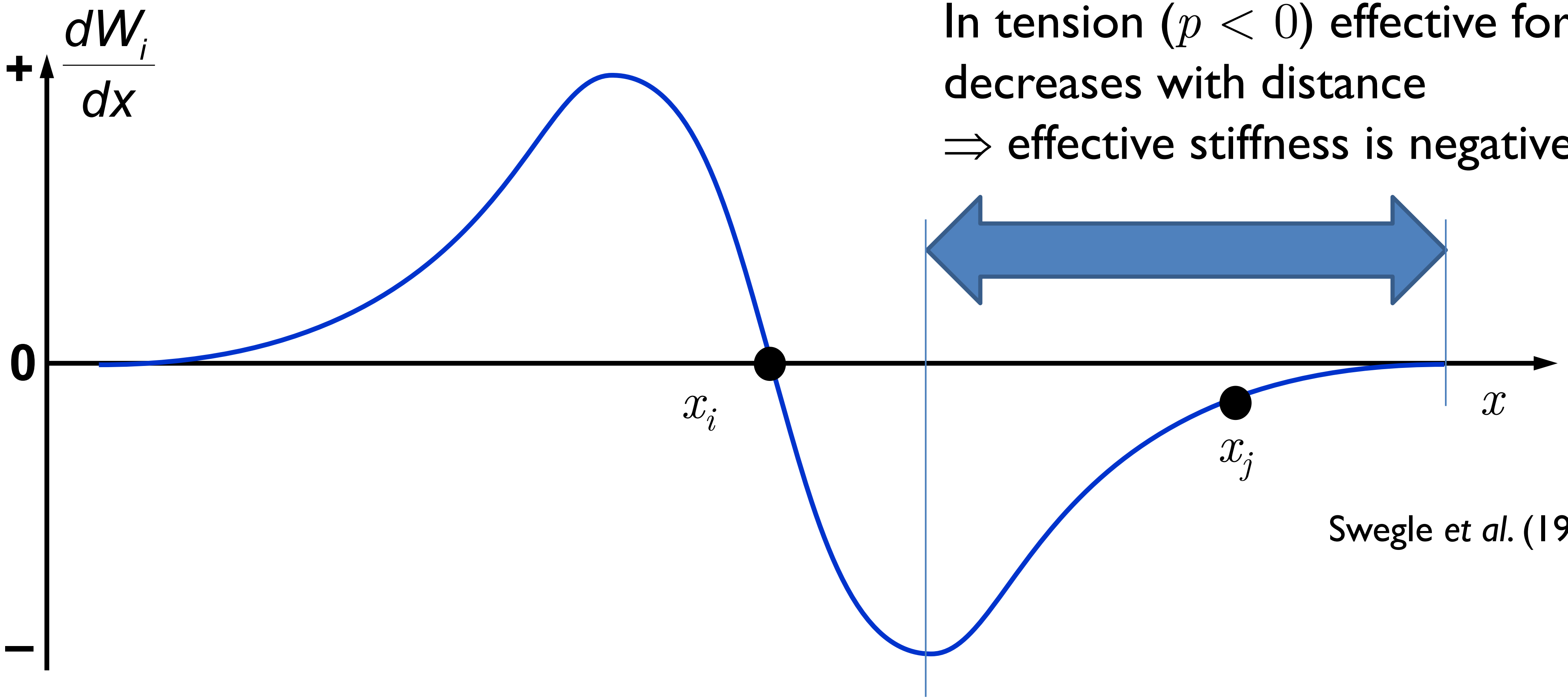
Tensile instability

Force on particle i due to j $\propto \; p_j \left.\frac{\mathrm{d}\,W_i}{\mathrm{d}x}\right|_{x=x_j}$

In this region, $W_i'' < 0$

In tension $(p < 0)$ effective force decreases with distance

$\Rightarrow$ effective stiffness is negative



Swegle *et al.* (1995)

Remedies for tensile instability

Repulsive forces (Monaghan, 2000)

- maintain particle distribution
- non-physical momentum source

Total Lagrangian (Rabczuk et al., 2004)

- kernel is a function of material coordinates
- particle retains a constant material coordinate **X** and constant kernel values in **X** space
- Unsuitable for large/unbounded deformation (fluids)

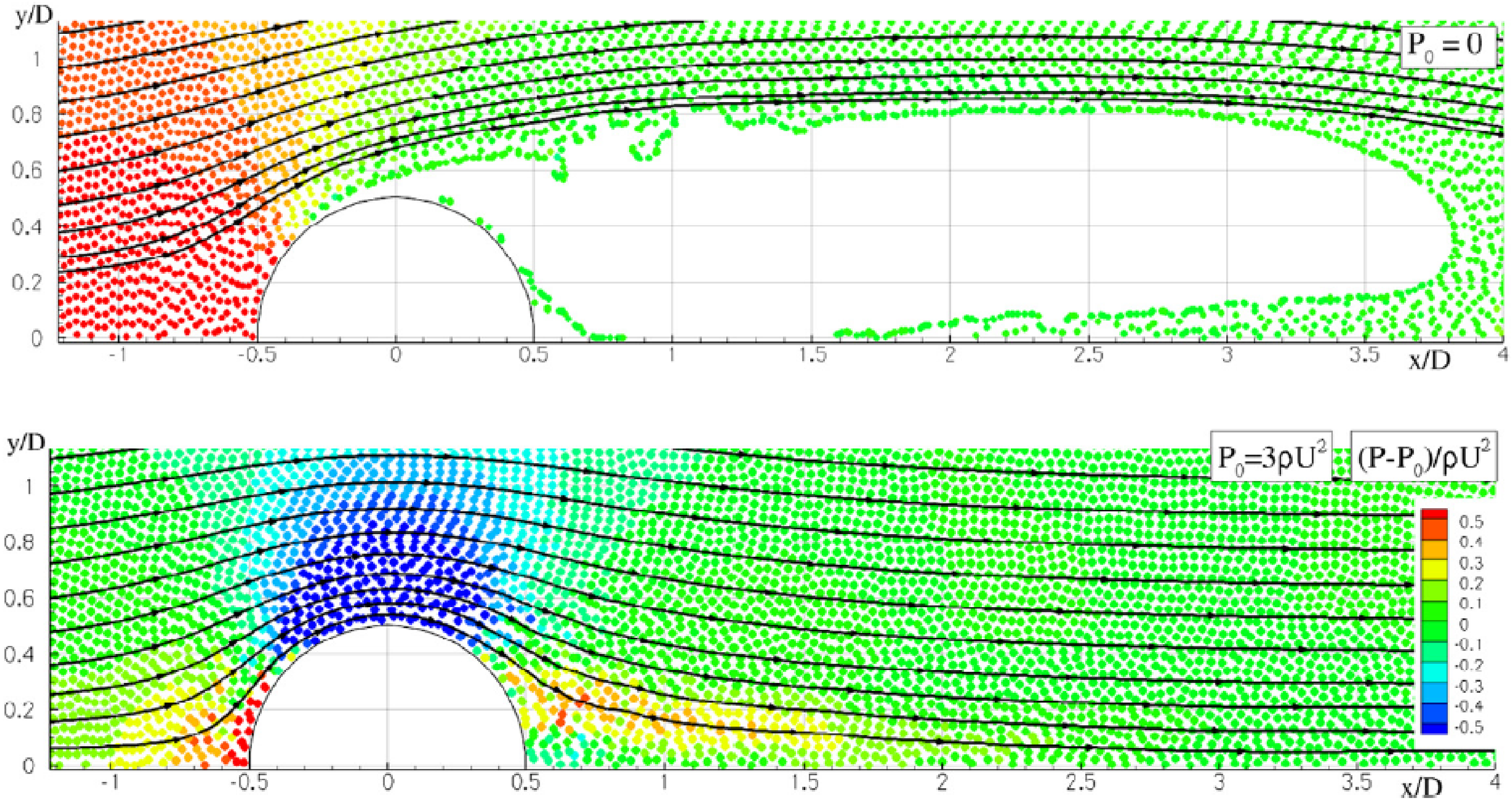
Non-collocational (Dyka and Ingel, 1995)

- stress and velocity stored at different points

Remedies for tensile instability

In fluids, simply avoid negative pressure!

e.g. add background pressure (but causes problems if too high…)

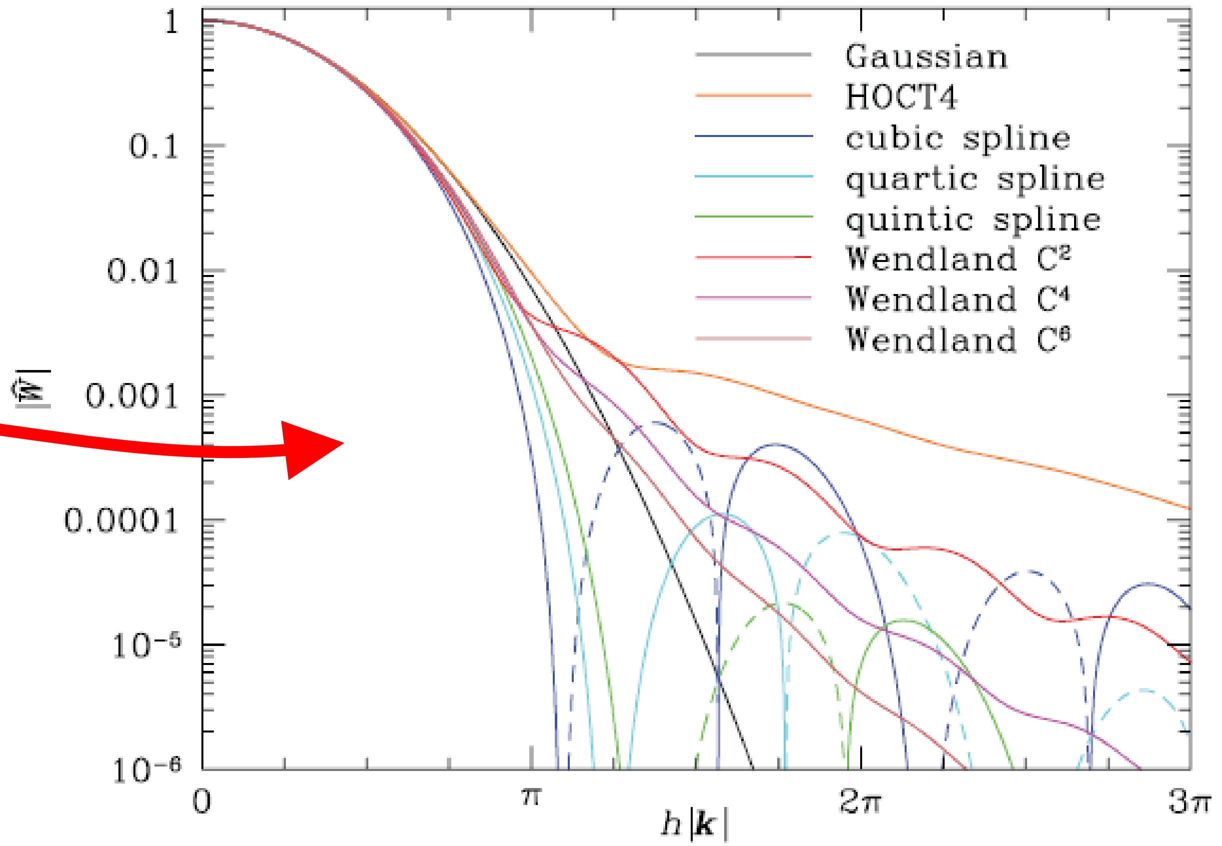


Marrone et al. (2013)

Pairing instability

If two particles are close, with $p > 0$, the repulsive force is small and gets smaller as they approach ($\nabla W \to 0$).

This is avoided if the kernel has non-negative Fourier transform everywhere – hence the Wendland kernels. (Dehnen and Aly, 2012)



SPH density equations

The continuity equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho \nabla \cdot \mathbf{u} = 0$$

SPH grad → $\frac{\mathrm{d}\rho_i}{\mathrm{d}t} = -\rho_i \sum_j \nabla W_{ij} \cdot \mathbf{u}_j \frac{m_j}{\rho_j}$

Alternatively:

$$\frac{\mathrm{d}\rho_i}{\mathrm{d}t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{u}) - \mathbf{u} \cdot \nabla \rho = 0$$

$$\frac{d\rho_i}{dt} = -\sum_j \nabla W_{ij} \cdot \left(\rho_j \mathbf{u}_j - \mathbf{u}_i \rho_j\right) \frac{m_j}{\rho_j}$$

$$= -\sum_j \nabla W_{ij} \cdot \left(\mathbf{u}_j - \mathbf{u}_i\right) m_j$$

SPH continuity equations

The summation density equation

Calculate density by interpolation (smoothing)

$$\rho_i = \sum_j W_{ij} \rho_j \frac{m_j}{\rho_j} = \sum_j W_{ij} m_j$$

Differentiate wrt *t*:

$$\frac{d\rho_i}{dt} = \frac{d}{dt} \sum_j W(\mathbf{x}_j - \mathbf{x}_i) m_j = -\sum_j \nabla W_{ij} \cdot (\mathbf{u}_j - \mathbf{u}_i) m_j$$

The two forms of density equation would be equivalent if integrated exactly (Vaughan *et al.*, 2008).

SPH momentum equations

$$\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}\boldsymbol{t}} + \frac{1}{\rho}\nabla p = 0$$

$$\frac{\mathrm{d}\mathbf{u}_i}{\mathrm{d}t} = -\frac{1}{\rho_i}\sum_j \nabla W_{ij} \cdot p_j \frac{m_j}{\rho_j}$$

Instead, use

$$\nabla\left(\frac{p}{\rho}\right) = \frac{1}{\rho}\nabla p - \frac{p}{\rho^2}\nabla\rho$$

This gives a commonly used version of the SPH momentum equation:

$$\frac{D\mathbf{u}_i}{Dt} = \sum_b \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2}\right)\nabla W_{ij} m_j$$

SPH momentum equations

Momentum equation:

$$\frac{d\mathbf{u}_a}{dt} = \sum_b \left(\frac{p_a}{\rho_a^2} + \frac{p_b}{\rho_b^2} \right) \nabla W_{ab} m_b$$

Force on *a* due to *b*: $\left(\frac{p_a}{\rho_a^2} + \frac{p_b}{\rho_b^2} \right) \nabla W_{ab} m_a m_b$

Force on *b* due to *a*: $\left(\frac{p_a}{\rho_a^2} + \frac{p_b}{\rho_b^2} \right) \nabla W_{ba} m_a m_b = -\left(\frac{p_a}{\rho_a^2} + \frac{p_b}{\rho_b^2} \right) \nabla W_{ab} m_a m_b$

The forces are equal and opposite if W_a and W_b are the same, thanks to symmetry of *W*

→ **Conservation of momentum**

We’ve examined consistency, and touched on stability and conservation.

Fresh interesting approaches to convergence:
Evers *et al.* (2018), Franz and Wendland (2021)

1. Consistency, stability, conservation, convergence
2. **(In)compressibility**
3. Walls
4. Some SPH variants

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Compressibility and incompressibility

Two choices for incompressible flow:

Incompressible SPH

- Pressure Poisson equation
- Implicit system
- Timesteps are short because of particle motion. Computational cost advantage is less than for mesh-based methods.

Weakly compressible SPH

- Equation of state for pressure as function of density
- Allows system to be fully explicit

Weakly compressible model

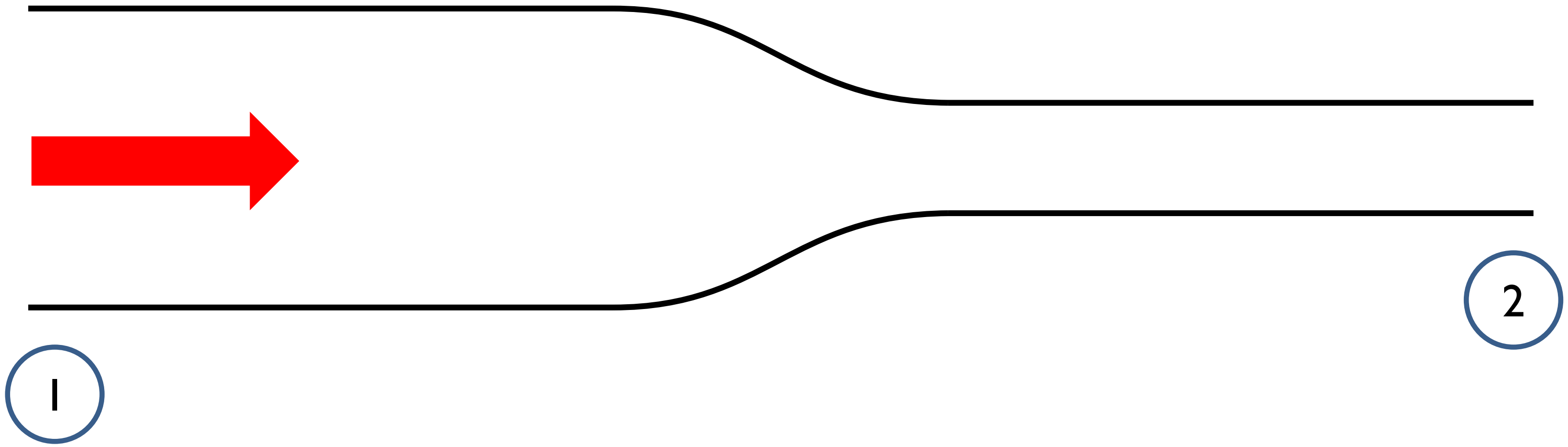
- Real fluids are somewhat compressible (finite speed of sound)
- WCSPH typically uses a much lower speed of sound c_0.
- Most use modified Tait equation of state:

$$p = \frac{\rho_0 c_0^2}{\gamma}\left[\left(\frac{\rho}{\rho_0}\right)^{\gamma} - 1\right]$$

or linearised version: $p = c_0^2\left(\rho - \rho_0\right)$

- Timestep is limited by Courant condition $\Delta t \le Ch / c_0$, with $C < 1$.
- We typically choose an unphysical low speed of sound c_0
- A common choice is $M = U / c_0 \cong 0.1$ for density variations ~1%.
- Why is this ok!?

Weakly compressible (WC) model



- Example: quasi-1D inviscid steady flow through a nozzle
- The exact incompressible solution satisfies Bernoulli:

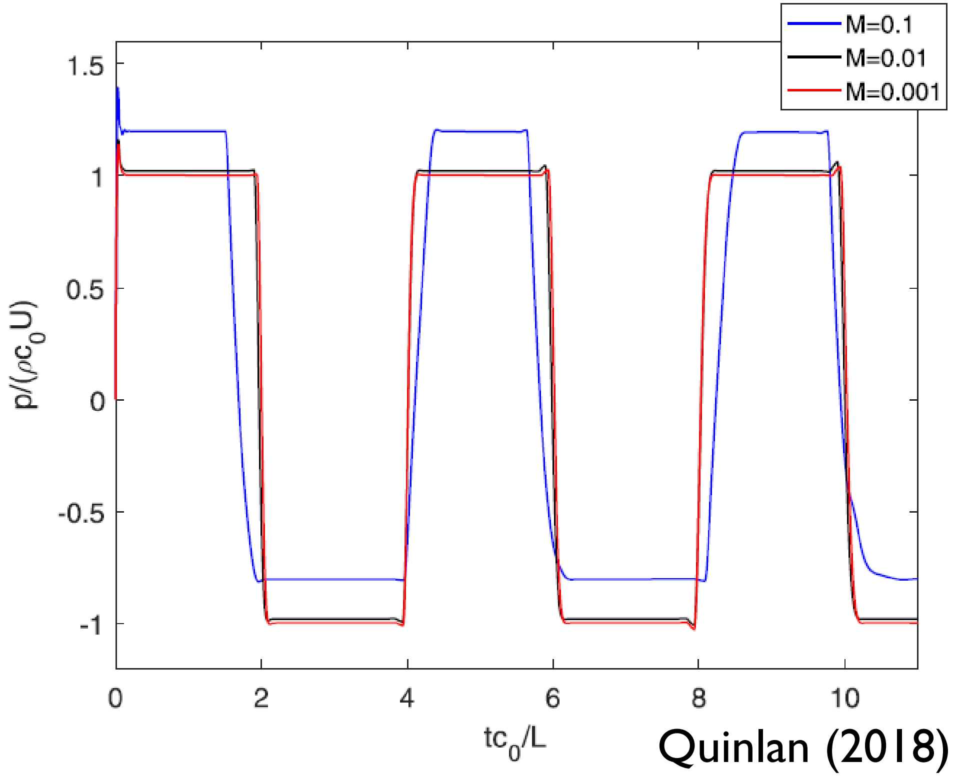
$$p_1 + \frac{1}{2}\rho u_1^2 = p_2 + \frac{1}{2}\rho u_2^2$$

- WC model can approximate this well if ρ is nearly constant – **speed of sound doesn't matter** in steady state…
- …or if underlying incompressible unsteadiness is slow (compared to acoustics)

Weakly compressible (WC) model

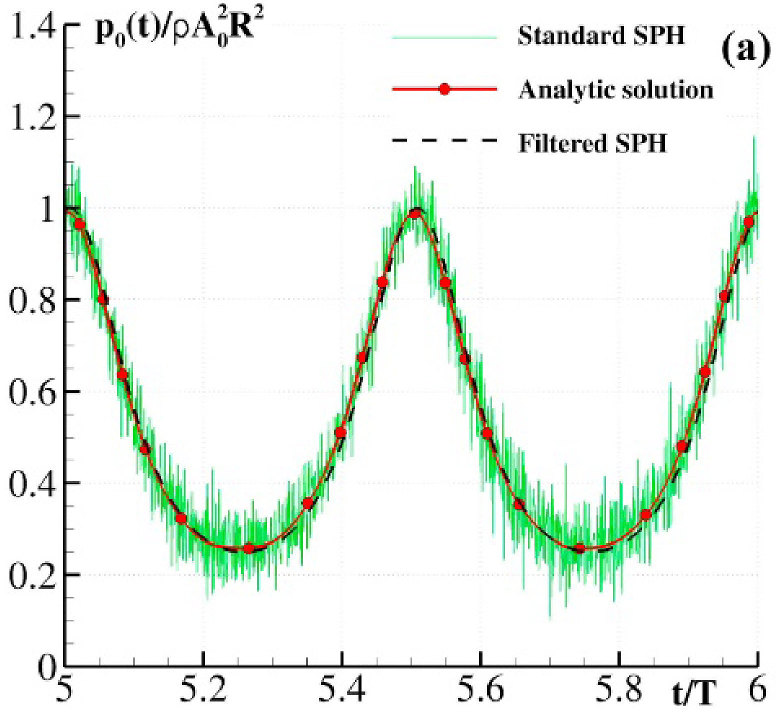
- "Acoustic" signals are a necessary part of the WC model.
- c_0 should be chosen so that acoustics are faster than mechanics
- Acoustics can even be filtered

Reflecting waves in a 1D impact event



Quinlan (2018)

Pressure in an oscillating drop



Meringolo et al. (2017)

1. Consistency, stability, conservation, convergence
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3. **Walls**
4. Some SPH variants

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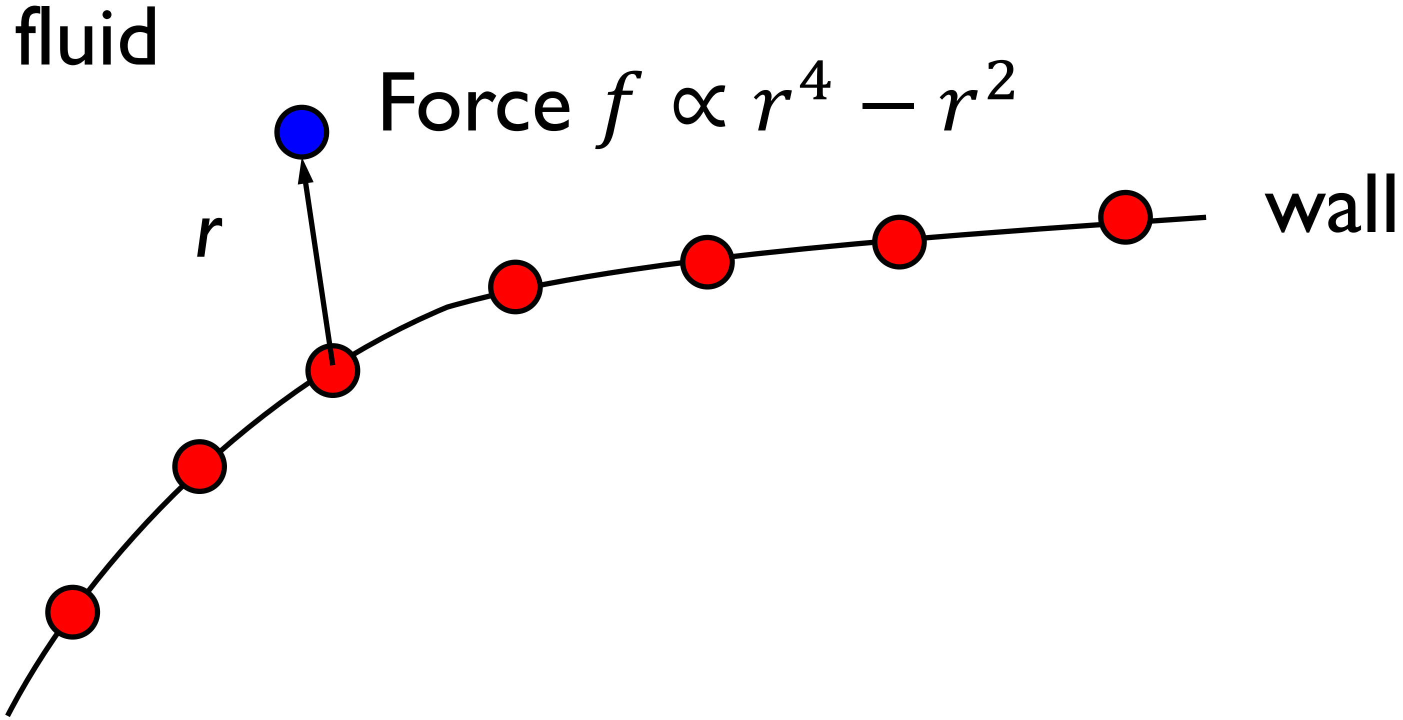
Wall boundary conditions in SPH

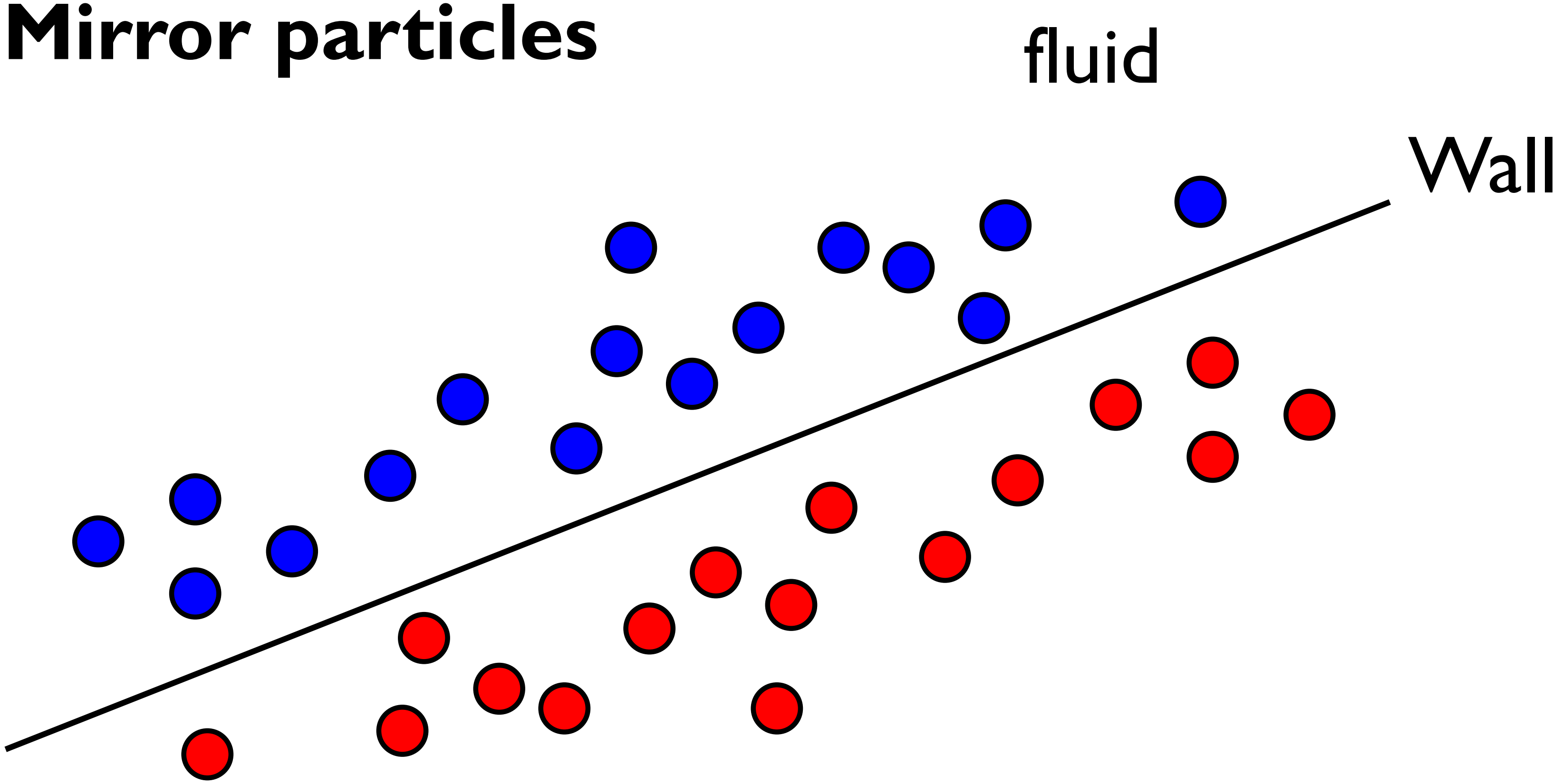
Repulsive wall particles



Monaghan (1994)

Boundary conditions in SPH

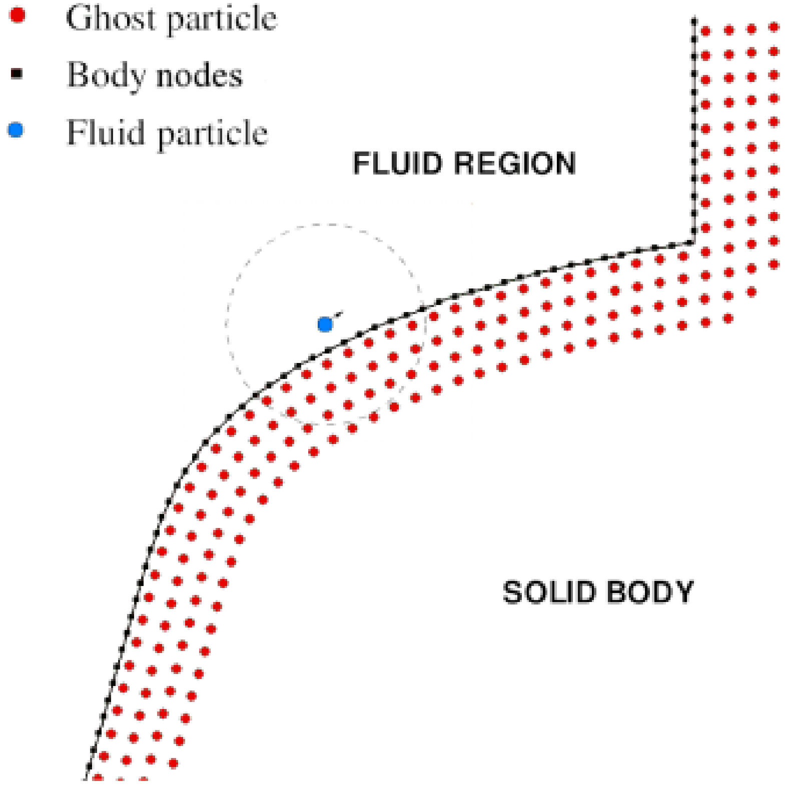
Mirror particles

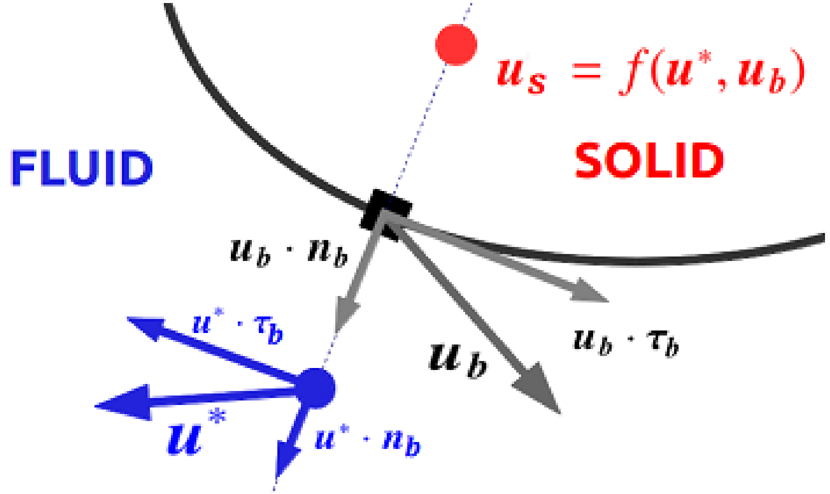


Boundary conditions in SPH

Ghost particles

Bouscasse et al. (2013)

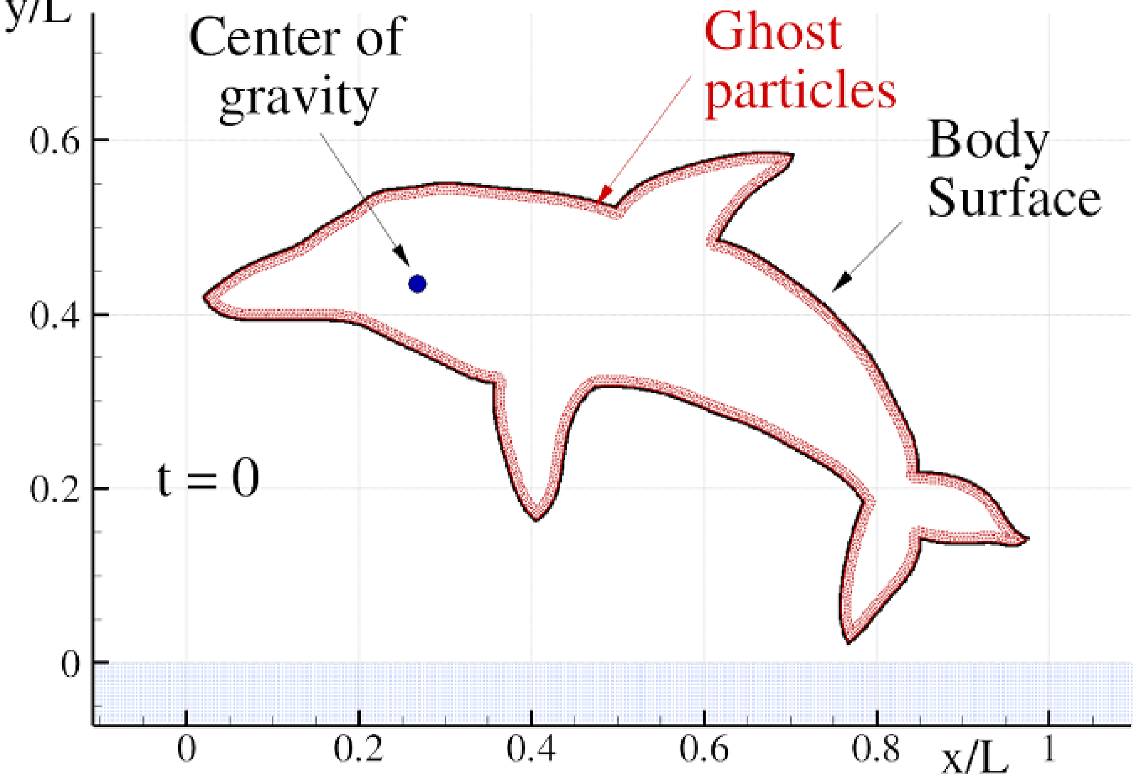


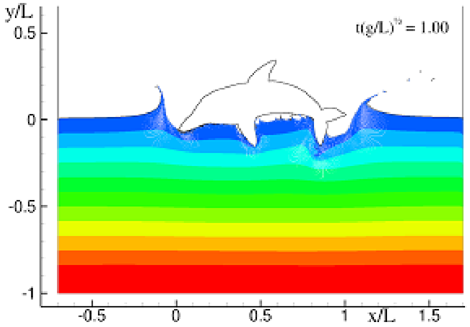


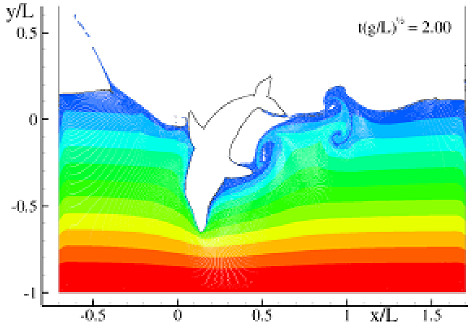
- Mirror **u,** p etc from fluid particles into wall
- Interpolate onto ghosts of ghosts in fluid

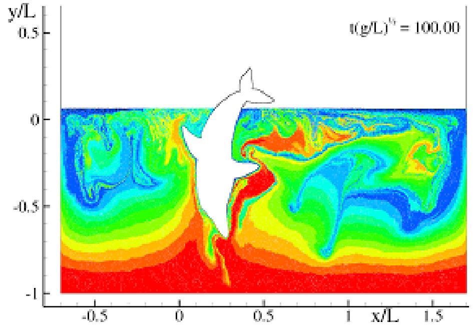
Boundary conditions in SPH

Bouscasse et al. (2013)





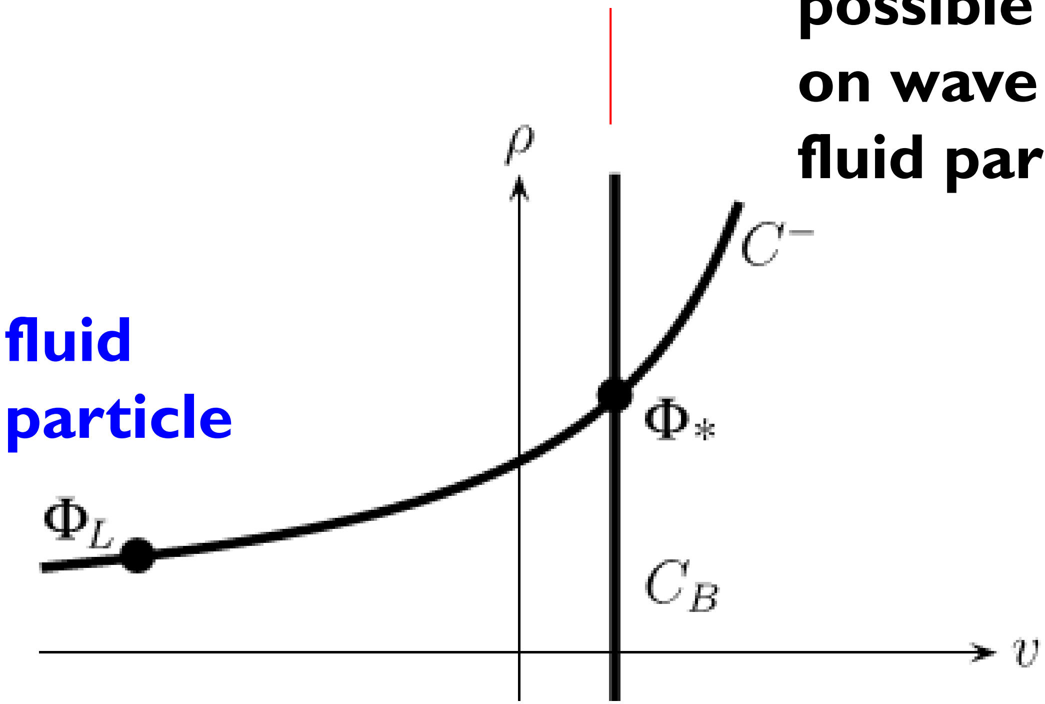




Boundary conditions in SPH

Partial Riemann Problems

Consider an acoustic wave from fluid particle to boundary (characterisitic wave, Riemann invariant)



wall prescribed velocity

possible states on wave from fluid particle

fluid particle

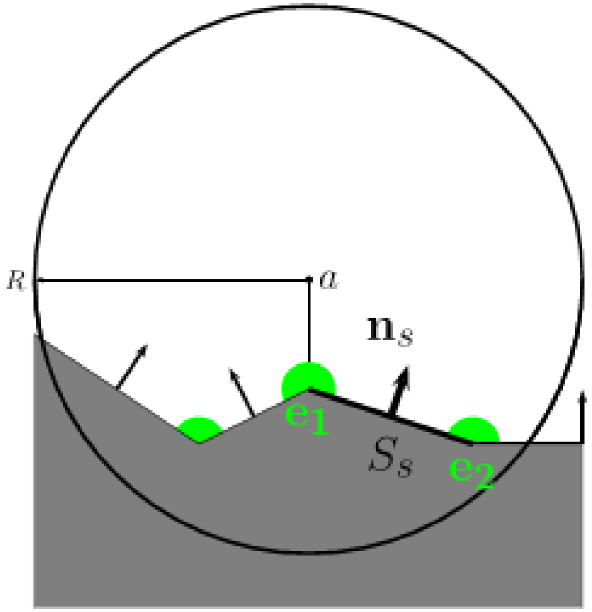
Marongiu et al. (2010)

Boundary conditions in SPH

Analytical

- Define volume normalisation near boundary
- Incorporate into SPH operators

$$\gamma_a \equiv \int_{\Omega\cap\Omega_a} w\left(\mathbf{r}' - \mathbf{r}_a\right) \mathrm{d}V'$$



$$\rho_a \simeq \frac{1}{\gamma_a} \sum_{b\in\mathcal{F}} m_b w_{ab}$$

$$\widetilde{\mathbf{Grad}}_a\{A_b\} \equiv \frac{\rho_a}{\gamma_a} \sum_{b\in\mathcal{F}} m_b \left(\frac{A_a}{\rho_a^2} + \frac{A_b}{\rho_b^2}\right) \nabla w_{ab} - \frac{\rho_a}{\gamma_a} \sum_{s\in\mathcal{S}} \left(\frac{A_a}{\rho_a^2} + \frac{A_s}{\rho_s^2}\right) \rho_s \nabla \gamma_{as}$$

Ferrand et al. (2013), Mayrhofer et al. (2014)
Kulasegaram et al. (2004)

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SPH variants: ALE-SPH / Riemann-SPH / Godunov-SPH

Apply SPH to conservative form of Euler equations

$$\frac{d}{dt}(\omega_i \Phi_i) + \omega_i \sum_{j \in D_i} \omega_j 2 G_E(\phi_i, \phi_j) \nabla_i W_{ij} = \omega_i Q_V$$

Particle volume

Conserved variables

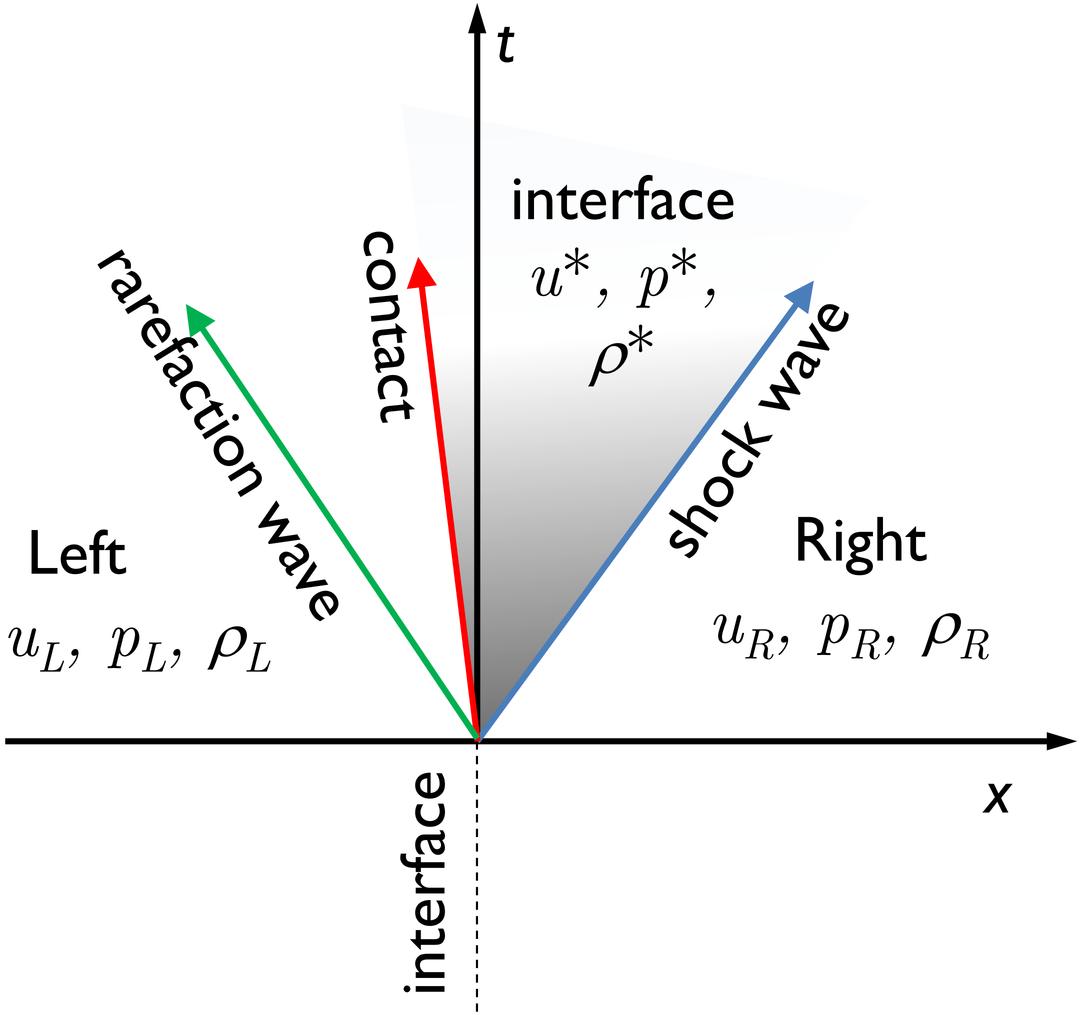
Godunov-type numerical flux function

The particles are now control volumes, exchanging mass.
“Particles” are not particles anymore!

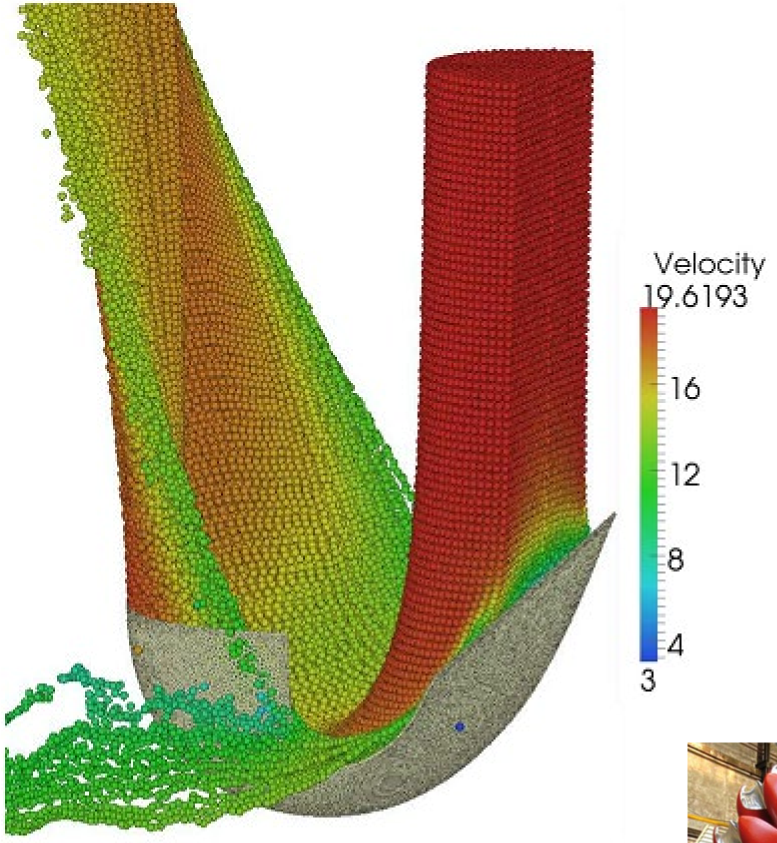
Vila (1999)
Marongiu *et al.* (2010)

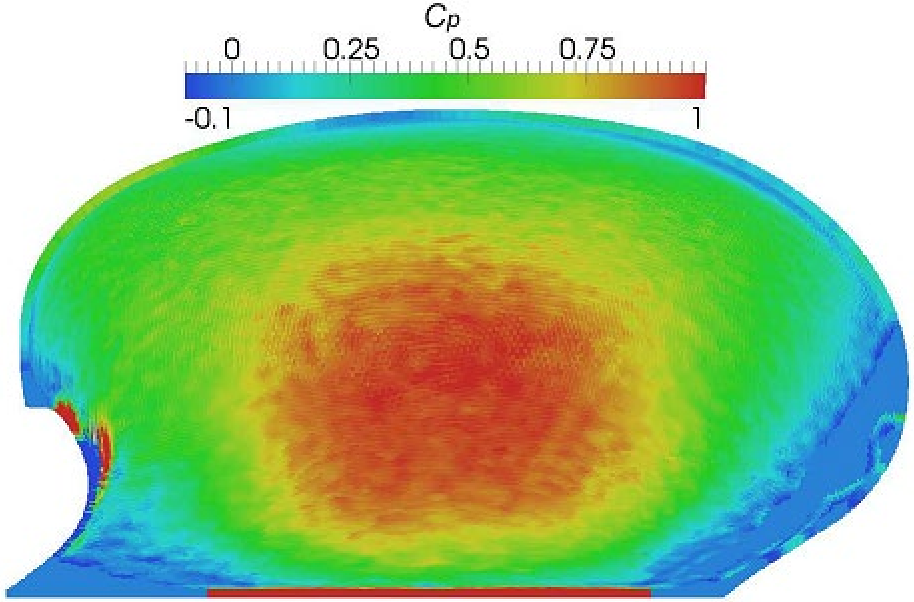
SPH variants: ALE-SPH / Riemann-SPH / Godunov-SPH

- Idea from compressible-flow finite volume methods
- At interface between particles/cells there is a discontinuity
- It generates 3 waves (sometimes simplified to 2) that are boundaries between 4 (3) uniform states
- We can use pre-cooked solutions to find the * state at the interface
- Do this at every particle-particle interface, at every timestep…



Pelton turbine





Marongiu *et al.*, 2010

SPH variants: δ-SPH

Dissipative terms of order *h*…
on *mass* as well as momentum

$$\psi_{ij} = 2(\rho_j - \rho_i)\frac{\boldsymbol{r}_{ji}}{|\boldsymbol{r}_{ij}|^2}$$

$$\frac{D\rho_i}{Dt} = -\rho_i \sum_j (\boldsymbol{u}_j - \boldsymbol{u}_i)\cdot\nabla_i W(\boldsymbol{r}_j)\,\mathrm{d}V_j + \xi h c_0 \sum_j \boldsymbol{\psi}_{ij}\cdot\nabla_i W(\boldsymbol{r}_j)\,\mathrm{d}V_j,$$

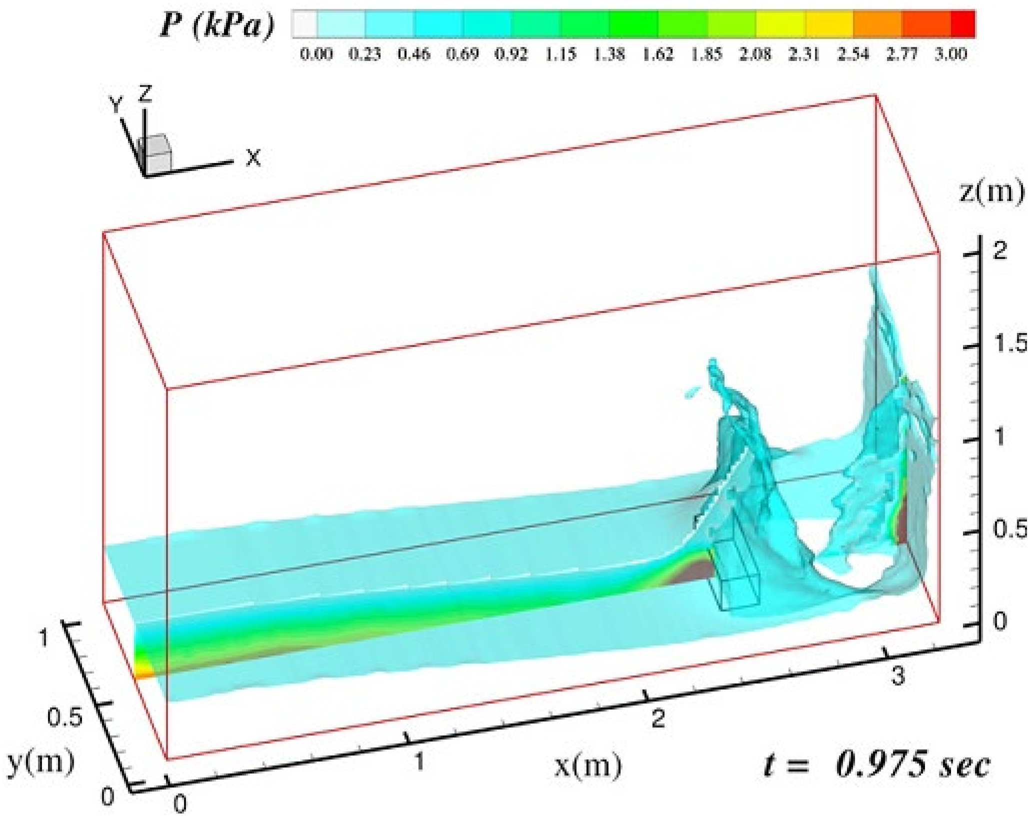
$$\rho_i \frac{D\boldsymbol{u}_i}{Dt} = -\sum_j (p_j + p_i)\nabla_i W(\boldsymbol{r}_j)\,\mathrm{d}V_j + \rho_i \boldsymbol{f}_i + \alpha h c_0 \rho_0 \sum_j \pi_{ij}\nabla_i W(\boldsymbol{r}_j)\,\mathrm{d}V_j,$$

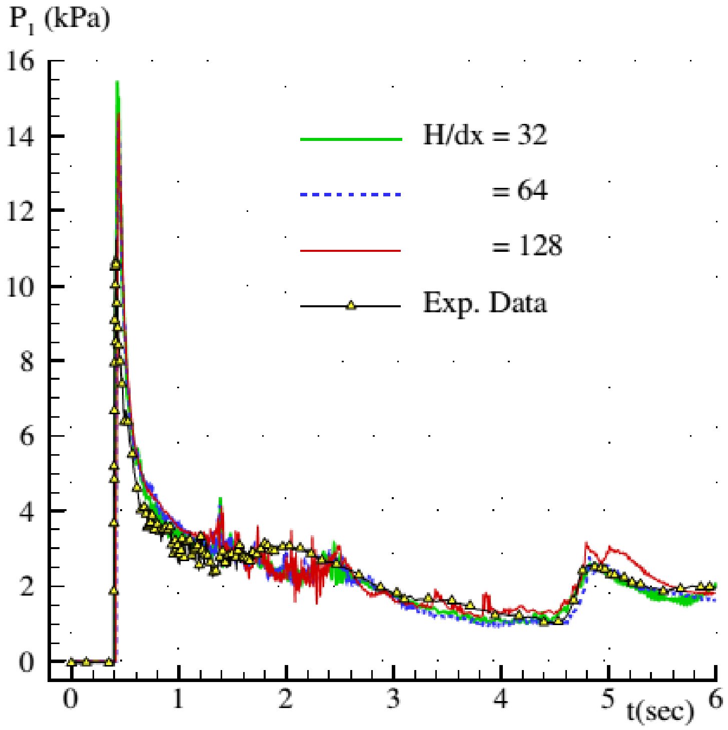
$$\pi_{ij} = \frac{(\boldsymbol{u}_j - \boldsymbol{u}_i)\cdot\boldsymbol{r}_{ji}}{|\boldsymbol{r}_{ij}|^2}$$

The new dissipation term converges to zero as *h* goes to zero.

Antuono et al. (2010)

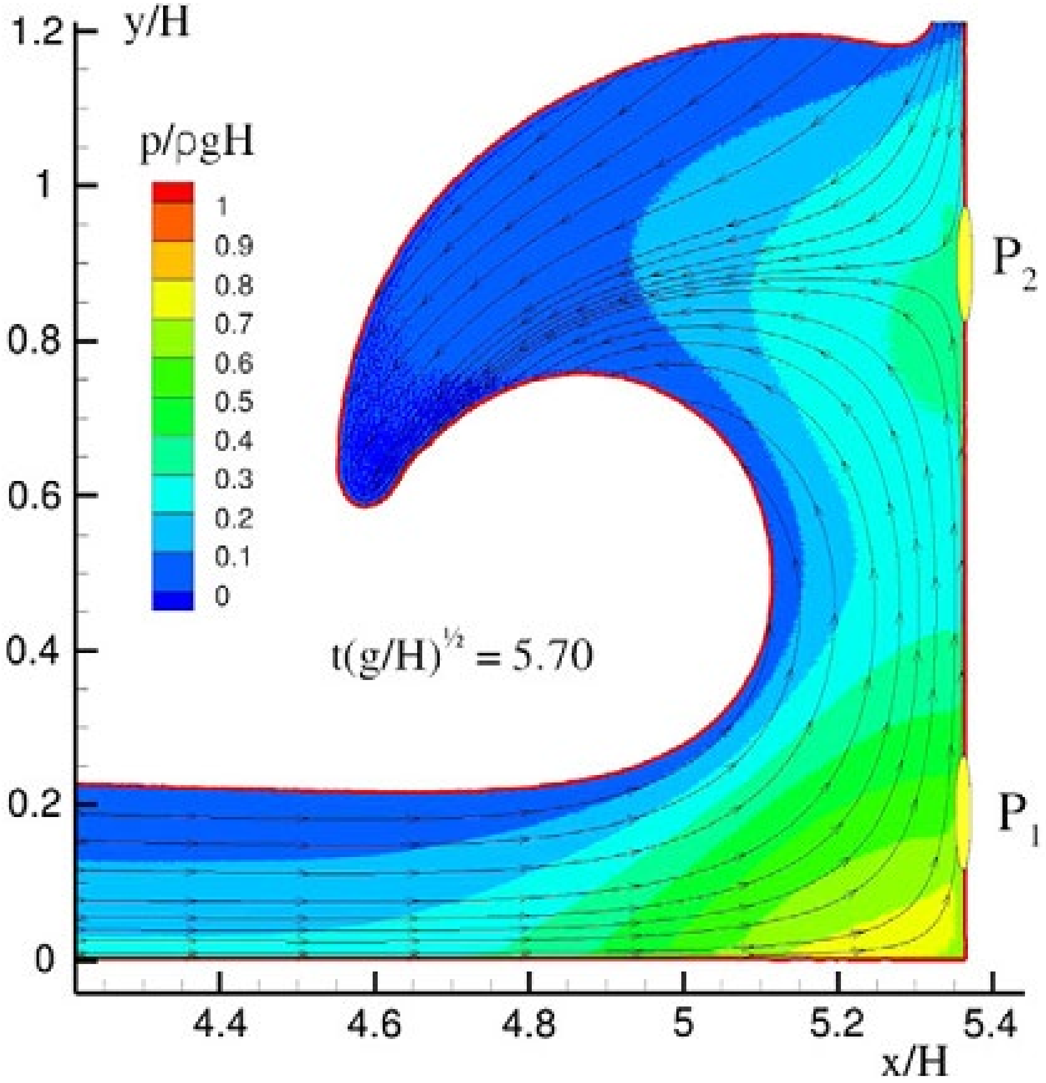
SPH variants: δ-SPH





Marrone et al. (2011)

SPH variants: δ-SPH



Marrone et al. (2011)

SPH variants: Finite Volume Particle Method (FVPM)

Conservation law:

$$\frac{\mathrm{d}\mathbf{U}}{\mathrm{d}t} + \nabla \bullet \mathbf{F}(\mathbf{U}) = 0$$

Introduce a test function $\psi_i(\mathbf{x})$:

$$\int_{\Omega} \psi_i \frac{\mathrm{d}\mathbf{U}}{\mathrm{d}t} \mathrm{d}\mathbf{x} - \int_{\Omega} \nabla \psi_i \bullet \mathbf{F}(\mathbf{U}) \mathrm{d}\mathbf{x} = 0$$

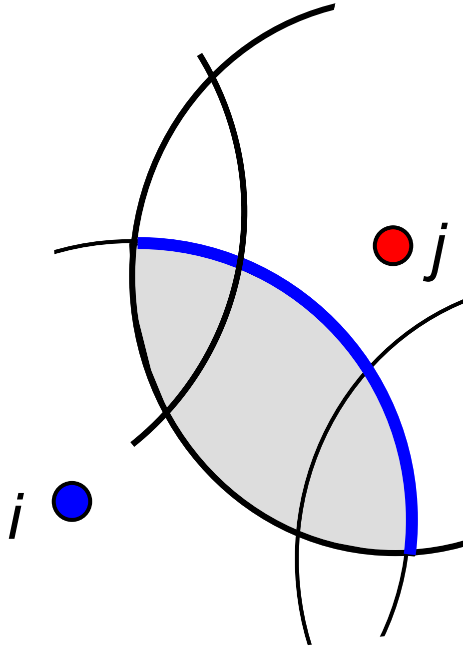
$\psi_i(\mathbf{x})$ is compactly supported and Shepard-normalised

$$\psi_i(\mathbf{x}) = \begin{cases} \dfrac{W_i(\mathbf{x})}{\sum_k W_k(\mathbf{x})} & x \in \Omega_i \\ 0 & \text{otherwise} \end{cases}$$

SPH variants: Finite Volume Particle Method (FVPM)

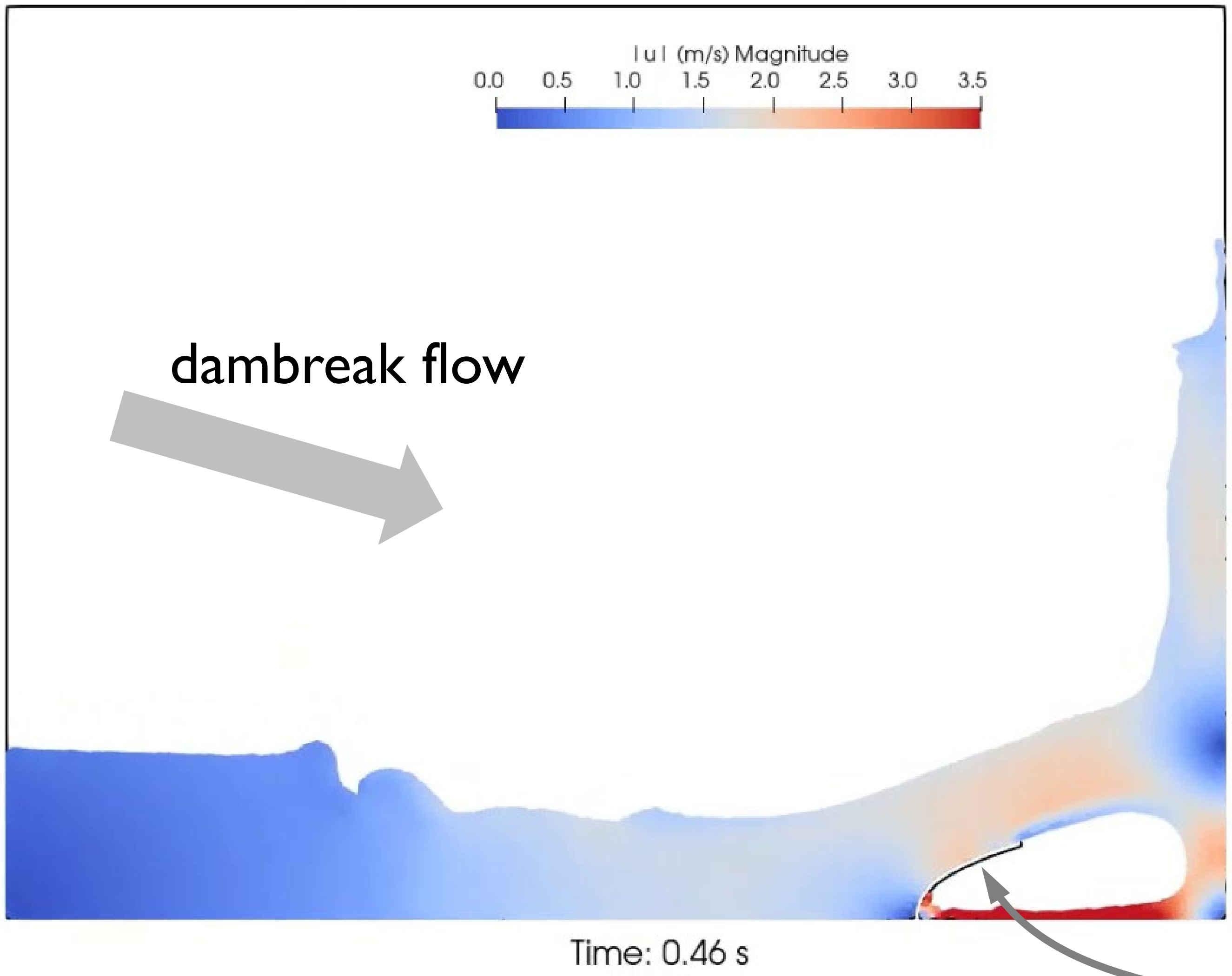
$$\int_{\Omega} \psi_i \frac{\mathrm{d}\mathbf{U}}{\mathrm{d}t}\mathrm{d}\mathbf{x} - \int_{\Omega} \nabla\psi_i \bullet \mathbf{F}(\mathbf{U})\mathrm{d}\mathbf{x} = 0$$

$$\beta_{ij} = \int \frac{W_i \nabla W_j - W_j \nabla W_i}{\left(\sum_k W_k(\mathbf{x})\right)^2} \mathrm{d}\,\mathbf{x}$$



- We have to integrate in the overlap of *i* and *j*
- Instead of using particles as quadrature points…
- …we do quadrature on new points in the overlap region…
- … or choose a special *W* to integrate fast and *exactly*

SPH variants: Finite Volume Particle Method (FVPM)



dambreak flow

elastic wall

Fluid-structure interaction test with FVPM and FEbio

McLoone et al. (2022)

We didn’t have time for…

- Particle shifting: giving the flow field some help to distribute particles nicely
- Energy conservation (without solving an energy equation)
- Pushing for higher orders of consistency

Reviews and recommended reading

Monaghan J.J. (1992) Smoothed particle hydrodynamics. Annual Review of Astronomy and Astrophysics 30:543–574.

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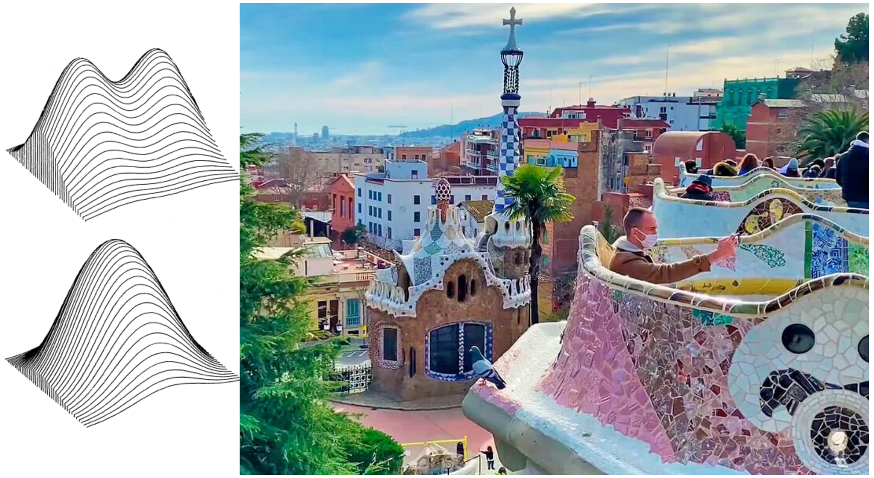
Another timeline

SPH

(my emotional journey)

This is so free and natural! I love it!

2002



oh... there is a lot to think about…

2003

…the work will never end…

2010

…I love it!

…